

MATHEMATICS AND REVOLUTIONARY THEORY: READING CASTORIADIS AFTER BADIOU

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ABSTRACT: The article offers a comparative analysis of the uses of set theory in Castoriadis's "The Imaginary Institution of Society" and Badiou's "Being and Event".

KEYWORDS: Ontology; Castoriadis; Badiou

The intellectual and political trajectory of Cornelius Castoriadis makes for a narrative of considerable complexity. It involves, to list only a few dramatic points, an early engagement in the communist party and a quick disengagement from it, a turn toward Trotskyism and a turn away from its own orthodoxies, and co-founding of the libertarian socialist group *Socialisme ou barbarie*, which for a brief time included the almost mythical figure of Guy Debord. All the while Castoriadis continued publishing on a broad range of subjects under several pen-names (Paul Cardan, Pierre Chalieu, Jean-Marc Coudray) in an effort to demonstrate that resistance required new ways of thinking and practical organization: a response to modern forms of capitalism and, at the same time, a critique of Marxism.

Castoriadis's emphasis on the revolutionary potential of diverse social groups, not only the proletariat, resonated well with parts of the student movement in France of the late 1960s. In an article from 1961, he argued that "the crisis of culture" is pushing students and intellectuals toward a radical critique of the system.¹ It was presumably for this reason that Daniel Cohn-Bendit, one of the leaders of the student revolt, regarded himself as a disciple of Castoriadis.² On the other hand, Castoriadis's assessment of Marxist theory was severe enough to be perceived as an attack, and could not have endeared him to a younger but later influential generation of philosophers who, for a time, were fascinated with Maoism.

It is not altogether farfetched, then, to imagine that broader political considerations

1. Castoriadis, *Socialisme ou barbarie* 6 (33) (December-February 1961), 84.

2. See: Daniel Cohn-Bendit, Gabriel Cohn-Bendit, *Obsolete Communism: The Left-Wing Alternative*, transl. by Arnold Pmerans (New York, McGraw-Hill, 1968).

should be reflected in Castoriadis's theoretical writings and in the reception of his work, even when the subject matter is as rarefied as the philosophy of mathematics. If true, this is perhaps especially true of *The Imaginary Institution of Society*.³ The book was composed over a period spanning a decade, from 1964 to 1974, and provides a picture of the development of Castoriadis's thought in a politically turbulent but extremely productive historical moment of French philosophy.

The text sweeps with verve and eloquence over the fields of philosophy, politics, psychoanalysis, history, science and—significantly for the purposes of this essay—mathematics. Although by no means the central concern in the book, mathematics plays an important role. Sets and the logic of sets, in particular, are somehow involved with ontology. The “ensemblist” logic, writes Castoriadis, “is equivalent to an ontological decision concerning what is and the manner in which it exists: what is is such that sets exist.” (IIS: 227).

Sets, logic, ontological decisions: terms familiar from the work of Alain Badiou, whose *Being and Event* argues that set theory, in a sense, *is* ontology. One might expect that Badiou would have taken an interest in the mathematical facet of Castoriadis's argument, as an object of critique or a foil if nothing else. But that may be too naïve. As far as I know, Badiou seems to have remained silent on this possibility. The name of Castoriadis does not appear, for example, in Badiou's précis of twentieth-century French philosophy.⁴ Indeed, I am aware of only one explicit reference in the vast expanses of Badiou's textual output: a marginal note in the entry on Jean-François Lyotard in the *Pocket Pantheon*, where a split within *Socialisme ou barbarie*, a break also between Castoriadis and Lyotard, is mentioned in passing.⁵

The article at hand is therefore best viewed an exercise in philosophical fiction; or, perhaps less provocatively, as an attempt at a comparative reading of (parts of) *The Imaginary Institution of Society* and (aspects of) *Being and Event* and related but less technical texts. The question I would like to address is not whether Badiou did in fact respond to Castoriadis, but whether the system of *Being and Event*—in hindsight and with a conscious nod to the speculative—may be seen *as* a critical reworking of Castoriadis's ideas on the relation of mathematics and ontology. There is, despite all the differences, at least this much in common to these projects: their ontologies involve the concept of set.

References to set theory and to the philosophical import of Georg Cantor do constitute a common ground, but they are also a source of fundamental disagreements. Castoriadis programmatically relied on naïve set theory, or something even less formal than that, whereas Badiou insisted on axiomatic formalization. Badiou was among the younger philosophers who in the years from 1966 to 1969 contributed to *Cahiers pour l'analyse* where, “guided by the examples of Althusser and Lacan, they sought to combine

3. Castoriadis, *The Imaginary Institution of Society*, transl. by Kathleen Blamey (Cambridge, Mass., MIT Press, 1987), 1-2. Further notes refer to this edition, abbreviated as IIS.

4. Badiou, “The Adventure of French Philosophy”, *New Left Review* 35, Sept-Oct 2005, 67-77.

5. Badiou, *Pocket Pantheon* (London, Verso, 2009), 104: “1964: the great split in the group, with Cornelius Castoriadis on one side, and the ‘Pouvoir ouvrier’ group, which Lyotard joined, on the other.”

structuralism and psychoanalysis with logical and mathematical formalization”⁶ Castoriadis, on the other hand, was an unsparing critic of structuralism and its ideal of formal rigor, and his relations with Lacan were complicated. It seems beyond doubt that part of the difference in the respective approaches of Castoriadis and Badiou stems from this clash of broader outlooks and reflects different allegiances forged in a struggle both philosophical and political.

Despite numerous sources of friction, it is worth emphasizing—if only to enable and set off this attempt at reading in the rearview mirror—that Castoriadis and Badiou are in their different ways addressing a political question of some importance to the Left of a certain era. Part of their considerable efforts is meant to overcome the problematic legacy of “scientific Marxism”, or the crude Engels-style “dialectics of nature”, by means of an appeal to mathematics as an *open* dialectic. Loosely speaking, mathematics, especially set theory, holds out the prospect of a science that does not rely on the deterministic paradigm of classical mechanics, a central image of the mechanical-Marxist approach to the “science of history”.

Castoriadis wants to do more than simply *modernize* Marxism. He means to put its method into question: “To say that being Marxist is being faithful to the method of Marx which continues to be true, is to say that nothing in the content of the history of the past 100 years either authorizes us or compels us to put Marx’s categories into question [...]” (IIS: 14) But the appeal to modern mathematics (as set theory used to be known in Europe) does have a modernizing aspect, and it is important to note that the new revolutionary theory retains connections to a paradigm of modern scientific rationality. It does not seek a return to the humanism of the young Marx⁷; it appeals to science but offers an image different from a clockwork universe, which metaphorically underpinned the scientific claims of mechanical Marxism.

Mathematical ontology evolves, it has a history, but it is a formal consequence of mathematics itself that its development involves making decisions about the formally undecidable. Such decisions cannot be reduced to rational determinations within a preconceived system of rules. A dialectic based on mathematics would therefore be *open*—on the level of metaphor at least—in the sense that it would leave room for incalculably creative acts and politically radical agents that cannot be derived from a priori “laws of history”. Mathematics thus enters the realm of revolutionary theory, though we have yet to see how exactly this is supposed to work.

The political question of the status of Marxism is where Castoriadis beings; the ontological role of set theory (or something like it) is revealed much later, some two hundred pages into *The Imaginary Institution of Society*. The first part of the book, “Marxism and Revolutionary Theory”, was published in *Socialisme ou barbarie* from 1964 to 1965,

6. Description from the web archive of the *Cahiers* project. The list of authors published in the journal is itself significant and includes Boole, Cantor, Russell, and Gödel, alongside Althusser, Lacan, Lévi-Strauss, Foucault, Derrida, and Badiou.

7. “Today’s sophisticated Marxists who only want to hear about Marx’s early manuscripts prove themselves to be both superficial and extremely arrogant, for their attitude amounts to saying that after the age of 30 Marx no longer knew what he was doing.” (IIS: 15)

and includes a description of closed dialectical systems along with an account of what is wrong with them. For a start: “A *closed* dialectic, like the Hegelian dialectic, is necessarily rationalist. It at once presupposes and ‘demonstrates’ that the whole of experience is exhaustively reducible to rational determinations.” (IIS: 54)

This holds for Marxism as well; and for Marx, although the young Marx is credited with having the right idea: “a transformation of the dialectic is possible, in its turn, only if the traditional and secular idea of theory as a closed system and as contemplation is superseded. And this was indeed one of the essential intuitions of the young Marx.” (IIS: 56) But it was only an intuition. Marx may have thought that Hegel’s dialectic had been on its head and that he put it on its feet, but in fact “matter and spirit in these philosophies are finally nothing but pure Being, that is, as Hegel precisely said, Nothingness.” (IIS: 55) Therefore, “a revolutionary surpassing of the Hegelian dialectic demands not that it be set on its feet but that, to begin with, its head be cut off.” (IIS: 55)

According to Castoriadis, Hegel speaks of spirit and Marxism of matter and of economic determinism (the notorious “determination in the last instance”), but there is no more than a terminological difference between Hegel’s “spiritualism” and Marxist “materialism”. It is worth quoting a longer excerpt which incidentally states Castoriadis’s position with respect to Mao, a significant issue in Paris of the 1960s and 1970s:

Calling oneself ‘materialist’ in no way differs from calling oneself ‘spiritualist’, if by matter is meant an entity otherwise undefinable, yet entirely subject to laws that are themselves consubstantial and co-extensive with our reason, and hence capable already now of being penetrated by us in principle (and even in actual fact, since the ‘laws of these laws’, the ‘supreme principles of nature and knowledge’ are henceforth known to us: these are the ‘principles’ or the ‘laws of the dialectic’ discovered 50 years ago and now even numbered thanks to comrade Mao Tse-tung). (IIS: 55)

A broad outline of a new dialectic is then given:

A ‘non-spiritualist’ dialectic must also be a ‘non-materialist’ dialectic, in the sense that it refuses to posit an absolute Being, whether as spirit, as matter or as the totality, already given in principle, of all possible determinations. It must eliminate closure and completion [...], seriously accept the idea that there is both the infinite and the indefinite [...]. (IIS: 56)

We can see, here, some ideas of interest to Badiou: incompleteness, openness, infinity, non-totalization, a “non-materialist” kind of materialist dialectics. Still missing, however, is the connection with mathematics. Before looking into how set theory comes up in *The Imaginary Institution of Society*, it is useful to set the book aside, temporarily, and attempt a very brief and very rough sketch of the strategic function mathematics has in *Being and Event*. It is easier to extract a first idea of the relation of mathematics to an open dialectic from Badiou’s work, which is more rigorous and more explicit on this point. This will also help contrast and compare his approach to that of Castoriadis.

One attraction of mathematics, more precisely set theory, as the basis of an ontology is that it precludes the positing of absolute Being, or at any rate can be seen as precluding

such an excess. There is no set of all sets—it leads to contradictions both in naïve set theory and in the axiomatic system Badiou chooses—hence there is no totality of Being, no ontological absolute. There is only a multiplicity of multiplicities, a plurality that never resolves itself into One; Being resists totalization, on mathematical grounds. Furthermore, mathematics is in a technical sense incomplete; it cannot be completed or closed, and hence fulfills this important condition of an “open dialectic”.

In Badiou’s case, the dialectic can also be described as a synthesis and critical overcoming of the views of his “masters”: Sartre, Althusser and Lacan. The problem is twofold. First, Althusser’s theoretical framework leaves little or no room for a free or revolutionary subject, or for genuine creativity. The synchronic structuralist view, which permits no eruption of historic events, was challenged in the streets of Paris in 1968. On the other hand, Lacan’s model of the subject is tied to psychoanalysis and applying it on a grander historical scale requires some adjustments; it was no less problematic politically, as suggested by Lacan’s unpopular message to the protesters, to the effect that “you are looking for a new master and you will find him”⁸.

Badiou retains some of Lacan’s insights and terminology, notably the concept of the real. There is a formally intractable real of mathematics, something that is beyond formalization yet makes formalization possible:

the mathematical consists of pure deduction. We always suppose that it contains no contradictions. But Gödel showed that it is impossible to demonstrate, *within* a mathematical theory, that this very theory is non-contradictory. A mathematical truth cannot *force* the non-contradictoriness of mathematics. We will say that non-contradiction is the *unnamable* of the mathematical. And it is clear that this unnameable is the *real* of the mathematical: for if a mathematical theory is contradictory, it is destroyed.⁹

However, Lacan’s concept of the real as an *impasse* of formalization undergoes a change in its passage from psychoanalysis to the history of science. From a broader historical viewpoint, the practice of mathematics offers a fair amount of evidence that an apparent impasse functions, in effect, as the daemon of formal innovation. For example, Gödel’s theorems and Russell’s paradox, though they appeared to be the moments of a grand “crisis of foundations”, were eventually absorbed into mathematics and became productive in an actual and practical sense.

Rather than being a dead end, an apparent impasse can become a transformative force. Over time, from the “scandal” of irrational numbers and the “impossibility” of taking square roots of negative numbers, to the present day and beyond, such transformations produce a series of strata, each necessarily incomplete. Thus the process

8. Badiou’s “first master”, Sartre, proved more relevant to the contemporary political situation, and experienced a revival after having been written off as “unscientific” during the structuralist reign. Reconciling Sartre and set theory is no small philosophical feat, but it is at best tangential to the subject at hand. It seems better to start from Badiou’s reworking of Lacan, whose enthusiasm for mathematics is well known.

9. Badiou, “Philosophy and Truth”, in: *Infinite Thought*, translated and edited by Oliver Feltham and Justin Clemens (New York, Continuum, 2005), 49-50.

continues with no closure and indeed no teleology, resulting in what Badiou calls “the history of formalization”. Encounters with various facets of the formally intractable real are precisely what drives the always-incomplete evolution of mathematics.

To sum up, in addition to the “negative” or limitative function of a mathematical ontology—no absolute Being, no completeness or closure—there is also a positive aspect: an apparent impasse, for example an encounter with a set-theoretic paradox, functions in practice as a “challenge” that can result in significant transformations. These are, Badiou writes, “singular moments when mathematics is called upon, for its own purposes, to think its thought.”¹⁰ This dual function, at once limitative and productive, is the open dialectic of mathematics.

Castoriadis does not attempt to infer the open character of mathematics from the incompleteness theorem, and mentions Gödel’s result only in passing. That mathematics is in some sense an open system is basically a postulate, supported by historical evidence to which Castoriadis alludes without overtaxing the reader, or himself, with too many details. Clearly he agrees that the “crisis of foundations” was not an impasse, and that mathematics, rather than collapsing into a heap, continued to evolve in a productive way: “Mathematics is obviously interminable, not only with respect to the proliferation of its results but also in terms of the substance of its ideas. In its case, no more than in others, we cannot think that its construction has been completed in the past 50 years; instead, this construction has exploded.” (IIS: 222)

Castoriadis and Badiou also agree that the truly significant moments are to be sought in the acts of making “ontological decisions” that are beyond formal justification. In Badiou’s case such a decision is related to what he calls an event, with “event” defined in a way that deliberately places it out of reach of ontology, that is, outside of set theory. An event is not a set; it is an irruption of Being from the beyond of (the present state of) the thought of being, hence unpredictable and incalculable. It requires an ontological decision, a new axiom and the corresponding ontological commitment (in the form of “fidelity”, since the decision has can have no formal justification).

For Badiou, these are transformative moments in the history of *formalization*, without which (namely: formalization) there can be no materialist ontology: “a truth begins with an *axiom of truth*.” For Castoriadis, on the contrary, the conclusion is that formalization is at best a secondary issue. It is the moment of genuine creativity that counts, and it comes from somewhere outside of what can be formalized. This creative moment may ultimately take the form of a new formalization, a new axiomatic system, but this is only a technical and in a sense derivative aspect of the process.

Before discussing the why and the how of this difference, let me emphasize an important and nontrivial point of convergence: Castoriadis and Badiou (of *Being and Event*) regard set theory as the basis of mathematics.¹¹ Thus Castoriadis:

10. Badiou, “Mathematics is a Thought”, in: *Briefings on Existence: A Short Treatise on Transitory Ontology*, translated by Norman Madarasz (Albany, SUNY Press, 2006), 51.

11. Nontrivial, because some mathematicians have taken exception to such a claim. Badiou also changed his mind some time in the 1990s, when he discovered topos theory.

we have seen a considerable unification of mathematics at the same time as an important elucidation of questions related to foundation. Both outcomes are essentially tied to the constitution and to the development of the theory of ensembles, set-theory, which today provides the language and the basic tools for all branches of mathematics, and for this reason, constitutes the first part of mathematics. (IIS: 222)

Set theory, the ensemblist logic, or the ontological thought that underlies the concept of set, provides the language and basic tools, and it is in this sense the “first part” of mathematics, but it is not a founding principle. It is itself a product of a much broader and indeed central concept of *The Imaginary Institution of Society*: the social-historical imaginary.

Castoriadis decouples his use of the term “imaginary” from the meaning it acquired in the Lacanian tradition and in the broader structuralist lore, and does so already in the Preface:¹²

it has nothing to do with that which is presented as ‘imaginary’ by certain currents in psychoanalysis: namely, ‘specular’ which is obviously only an image *of* and a reflected image, in other words a *reflection*, and in yet other words a byproduct of Platonic ontology (*eidolon*) even if those who speak of it are unaware of its origin. The imaginary does not come from the image in the mirror or from the gaze of the other. Instead, the ‘mirror’ itself and its possibility, and the other as mirror, are the works of the imaginary, which is creation *ex nihilo*. (IIS: 3)

The notion of “creation *ex nihilo*” by no means makes it easier to understand the new meaning of the term “imaginary”. The depiction should perhaps be understood as a hyperbole intended to set free those who are still tied up with Plato’s allegory of the cave. The imaginary is not mere reflection, it is a truly creative force:

Those who speak of ‘imaginary’, understanding by this the ‘specular’, the reflection of the ‘fictive’, do no more than repeat, usually without realizing it, the affirmation which has for all time chained them to the underground of the famous cave: it is necessary

12. Through recombinations of Marxism and structuralism in the 1960s, the term “imaginary”—which in Lacan’s schema evokes a narcissistic demand, a fantasy of completeness or totalization, as well as fetishism and alienation—acquired the connotations of the Marxian term “ideology”. Indeed, Castoriadis charges the imaginary with producing representations, an ideological function par excellence in the view of some of his contemporaries. Not much is said about this issue in text, although Castoriadis notes that he has dealt elsewhere with “platitudes proffered about ‘epistemological breaks’” (IIS: 273); which presumably refers to Althusser or Bachelard. A number of unnamed French thinkers who questioned the relation of science and ideology and worried about the ideological function of representation are collectively dismissed in an amusingly vitriolic endnote. Castoriadis here implies that the radical Parisian intellectuals of the 1960s and 1970s read German philosophy in the famously slapdash fashion of Madame de Staël: “The term ‘representation’—employed by Freud almost as many times as the number of pages in his writings—lends itself to debate to the extent that it leads one to understand that ‘that which’ is posited in and through representation represents something else (*Vertretung* in German). The German word *Vorstellung* (from *vorstellen*, ‘put’, ‘pose’, ‘place before’) would be less liable to mislead; this did not prevent Heidegger from denouncing it as a manifestation of the modern tendency ‘to forget Being’, in various texts which, whether they have actually been read or not, continue to produce devastating effects among cultivated Parisian ladies who have taken to hold representation with abhorrence.” (IIS: 400n2)

that this world be an image *of* something. The imaginary of which I am speaking is not an image *of*. It is the unceasing and essentially *undetermined* (social-historical and physical) creation of figures/forms/images, on the basis of which alone there can ever be a question *of* ‘something’. What we call ‘reality’ and ‘rationality’ are its works. (IIS: 3)

Several questions now arise, and they relate to mathematics. The most obvious one is that of the status of Castoriadis’s assertions, namely of the “Archimedean” point from which he could make the statement that “what there is” is universally produced by a social-historical imaginary. Castoriadis tries to get around it in the Preface by claiming that he is not offering a *theory* but an “elucidation”. The disclaimer seems slightly disingenuous given the hundreds of pages that follow it, and in any case an elucidation of how reality and rationality are constructed-in-general might be expected, by the very logic of such an argument, to include an element that prevents the descent into relativism (sociohistorical, cultural, linguistic). If this universal element is the ontological thought that underlies set theory—as much a creation of the social-historical imaginary of nineteenth century Germany as Hegel’s “spiritualism”—why should it have universal relevance to the world-making ways of all societies?

In a different form, this is an important issue for Badiou, and one of the functions that mathematics has in his system is to serve as a shield against relativism. As ontology it is supposed to provide—rigorously formalized and historically variable but unmediated—access to what can be thought of being *qua* being. It is thus meant to relieve the symptoms of the linguistic turn, of an unbounded hermeneutics where everything is a matter of point of view, subject-position, or reader-response: “Mathematics has the virtue of not presenting any interpretation. The Real does not show itself through mathematics as if upon a relief of disparate interpretations”.¹³

It is not entirely clear (to me) whether Badiou was able to achieve the ambitious goal of overturning the linguistic turn, but it seems that already Castoriadis had faced some of the difficulties. They are compounded in his case by the traditionally materialist position—in this respect still Marxian—that every thought, including ontological thought, is a product of social-historical practice: “Every thought, whatever it may be and whatever may be its ‘object’, is but a mode and a form of social-historical *doing*.” (IIS: 3).

This holds for mathematics too, and hence there is a crucial ambiguity in Castoriadis’s descriptions of its ontological function. Ontology, on the one hand, must have a history, being itself a product of social-historical imagining and doing; on the other hand, it contains an invariant principle. The social-historical imaginary is differently instituted in different social and historical conditions, but contains a universal element common to all its particular actualizations. This universal element is, essentially, the kind of thinking that operates in set theory, and indeed in all ontological thought (which simply achieved a particular clarity in the theory of sets).

Badiou’s system is similar in this respect. There is, of course, a history of formalization, that is, the history of ontology. But at the time of *Being and Event* Badiou (like Castoriadis)

13. *Briefings on Existence*, 56.

maintains that set theory suffices to formulate all of mathematics, so that all conceivable ontological statements are already implicit in set theory. Hence the framework of sets can be taken as a “higher logic” in which to analyze the history of ontology. Here is how Castoriadis explains it: “The logical rudiments of set-theory are important in this respect for, regardless of what may happen in the future from the perspective of mathematics, they condense, clarify, and exemplify in a pure manner what, all the while, was underlying identitary logic [...]” (IIS: 222-3)

There are no ultimate “laws of history” here or “laws of the dialectic” in the style of Mao. Set theory is merely an abstraction of a type of logic that operates in the ontological thought of any social-historical institution. This “ensamblist” logic is what Castoriadis calls *legein*: the operation of “distinguish-choose-posit-assemble-count-speak”. Something like the operation of collecting elements into a set, naming the set and positing its existence, in keeping with the conventions of mathematical writing: “let X be...”. In this sense Castoriadis’s *legein* resembles the operation that Badiou, in *Being and Event*, calls “count-as-one”.

Actually, Castoriadis gives a number of different descriptions but no formal definition of *legein*, so it is not easy to pinpoint the exact meaning. For instance, an important additional description states that “[t]he central operation of *legein* is *designation*. Already ‘this is called...’ fully involves the entire bundle of operators that we normally think of separated and separable.” (IIS: 244) Furthermore, *legein* cannot be separated from another aspect of the social-historical imaginary, which Castoriadis calls *teukhein*, a sort of *techné*, a principle of action, organization, purposeful *doing*, described as “assembling-adjusting-fabricating-constructing” (IIS: 260).

Every logic requires technique, and every technique is a kind of logic; *legein* and *teukhein* thus form a true dialectical unity. Taken in separation, they yield two familiar (and misguided) extremes: “*Legein* [...] becomes at the limit the incoherent and untenable fiction of a purely formal system closed upon itself. *Teukhein* [...] becomes in the same way the incoherent and untenable fiction of technique for the sake of technique.” (IIS: 264) Due to its circular (or dialectical) relation with social-historical doing, the ensamblist logic that underpins *legein* “is potentially the means of an unlimited opening onto what, at the start, was not taken into account in its organization.” (IIS: 269).

The logic of sets is rudimentary and (supposedly) easy to understand, but on it rests, among other things, the ceaseless opening that is mathematics. Hence, in particular, *legein* has a history: “beyond this invariance of its abstract mode of operation, *legein* is at once subject to historical alteration and an active instrument of this alteration.” (IIS: 271) Let me confirm this very important point of convergence with Badiou by a longer quote, before irreconcilable differences emerge:

Leaving aside the invariance of the general type of organization of the *legein* mentioned above, there is, to be sure, a vast history of *legein* as *legein* in the broadest sense, which, considered cursorily from our contemporary perspective, appears as a ‘progressive’ evolution just as important as that of the productive technique in the narrow sense [...]. (IIS: 271)

That sounds a bit like what Badiou calls the history of formalization, and it is placed here on a par with the history of modes of production. It is tempting, if only for a moment and to get a sense of Castoriadis does *not* want, to compare this with a statement from Badiou's *The Concept of Model*:

We mustn't lose sight of Lacan's fundamental theses regarding the materiality of the signifier. By their light, Bachelard's celebrated definition of scientific theories as 'materialized instruments' rightly applies to these scriptural apparatuses that are formalized syntaxes: syntaxes that are in all reality *the means of mathematical production*, by the same right by which we may call vacuum tubes and particle accelerators the means of production proper to physics.¹⁴

A major difference surfaces now. It concerns "the means of mathematical production": formalized axiomatic systems (according to Badiou, or Bachelard whom he cites). Particularly important is the question of formalization of set theory, which places a priori limits on the scope of existential claims. To locate the source of the disagreement, let us go back to the point where Castoriadis first introduces sets: "These rudiments, indeed, posit and constitute explicitly both the type of logic, in its greatest generality, required by identity logic and the relations necessary and *almost sufficient* for this logic to function *unhampered and without limit* [...]" (IIS: 223, emphasis added)

Despite the rhetorical device of the artfully placed "almost", it is impossible to reconcile the "unhampered" and "without limit" with a consistent logic of sets: set-theoretic paradoxes impose a boundary. The naïve definition of sets, especially when used outside of the disciplinary norms of mathematical practice, amounts to a license to conjure up beings out of mere language. Even within mathematics, as shown by the well-known paradox discovered by Russell in Frege's *Grundsetze*, thinking of sets as "objective correlates" of linguistic constructs can lead to paradoxes. However, Frege is mentioned only once, parenthetically, and no detailed discussion of Russell's paradox can be found in *The Imaginary Institution of Society*.

Castoriadis relies on the logic of sets as an ontology, but also wants something more than the "ensemblist" logic formally grants; formalization stands in his way. A crucial departure from mathematics occurs when he writes: "The proof of the existence of God for a given society is the existence in its language of the word 'God'" (IIS: 271) In mathematics, naming does not by itself constitute an ontological proof; even Cantor, whose meditations did include a prominent theological element, had to heed paradoxes and to look for God in "inconsistent" or "absolute" infinities of collections that are too large to be sets.

There is, admittedly, a solid practical reason for Castoriadis's attempt to include such entities: "spirits, gods, myths and so on do *exist* for society." (IIS: 234) Furthermore: "For a society, to say that a term *is* means that it signifies (is a signification, is posited, is tied to a signification)" (IIS: 235). One might argue, by analogy, that sets exist in precisely the same sense for the Canadian Mathematical Society: sets *are* because they

¹⁴ *The Concept of Model*, ed. and transl. by Zachary Luke Fraser and Tzuchien Tho (Melbourne, re.press, 2007), 43.

signify and have significance. And it is only fair that other societies be granted the same ontological freedom. Yet in an ontology so inclusive, it is not clear on what grounds to exclude Hegel's absolute concept: "the simple essence of life, the soul of the world, the universal blood, whose omnipresence is neither disturbed nor interrupted by any difference [...]"¹⁵ It may be that these entities are socio-historical constructs, social fictions as it were, and acceptable as such. The ensemblist logic does not suffice to account for them, and indeed the working of the social-historical imaginary cannot be reduced to logic: "To be sure, it would be a critical mistake, a murder of the object—the structuralist murder—to claim that this logic exhausts the life, or even the logic, of a society." (IIS: 228).

But then it is not clear why Castoriadis spends a fair bit of time explaining the special role of sets and ensemblist logic. At best, sets may be an example of socially imagined objects, an example that has the virtue of mathematical clarity, but no different in principle from "the social contract" and "the bald king of France". At worst, it would seem that the logic of set theory, liberated from the technical requirements of consistency, could in some sense justify the Hegelian excesses that were supposed to be decapitated in the open dialectic: if *legein* operates unhampered by set-theoretic paradoxes, then there is nothing to stop it from positing the existence of an absolute, not only the "relative absolute" of a socially constructed God, but the "absolute absolute", the One. However, in a different part of the text Castoriadis is critical of theological fantasies: attacking the idea of the finitude of man, he writes that it "can acquire meaning only in reference to a theological phantasy and its translation into a philosophical thesis on God's infinity" (IIS: 199). Thus there seems to be an ambivalence here, only in part related to the question of sets.

It is instructive to consult Badiou, who appears to be a more attentive reader of Cantor. In Meditation Three of *Being and Event*, he writes that Cantor was "essentially a theologian" but deserves philosophical credit for helping to formulate a properly mathematical ontology. It was in his wavering between mathematics and theology, in his tortured compromise between "consistent" and "inconsistent" infinities, that Cantor anticipated the dividing line between a mathematical, non-totalizing thought of Being, and onto-theology that even at the price of paradox wants to include an inconsistent but absolute One (the troublesome "set" of all sets):

That it be in the place of this non-being that Cantor pinpoints the absolute, or God, allows us to isolate the decision in which 'ontologies' of Presence, non-mathematical ontologies, ground themselves: the decision to declare that beyond the multiple, even in the metaphor of its inconsistent grandeur, the one is.¹⁶

Badiou infers that a non-theological, mathematical ontology requires an axiomatic formalization which places a restriction on certain kinds of existential statement. In the Zermelo-Fraenkel system, the chosen framework of *Being and Event*, the restriction

15. *Phenomenology of Spirit*, transl. by A. Miller (Oxford, Oxford University Press, 1977), 100.

16. Badiou, *Being and Event*, translated by Oliver Feltham (New York, Continuum, 2007), 42. Further notes refer to this edition (paperback), abbreviated as BE.

on the conjuring powers of language is known as the Axiom of Separation: a formula can induce the existence of a set it describes, but only in the limited context of an already existing set. It is for this reason that Badiou, in one of his meditations on the axioms of set theory, writes that “[l]anguage cannot induce existence, solely a split in existence”. This is what permits him to think of axiomatic set theory as the basis of a *materialist* ontology: “Zermelo’s axiom is therefore materialist in that it breaks with the figure of idealinguistry—whose price is the paradox of excess—in which the existential presentation of the multiple is directly inferred from a well-constructed language.” (BE: 47)

Castoriadis, however, seems undeterred by paradoxes. The central ontological operations of *legein* are something that goes beyond all theory: they are “the ensemblizing operations without which set theory, the *theory* of sets (whether naïve or not), cannot even commence.” (IIS: 241). A fortiori, then, working with an axiomatic formalization puts us at a remove from the fundamental ontological insight. Since this insight is best described by Cantor—not perfectly, because Cantor’s is a *theory* of sets, but still—we must go back to the “naïve” concept, precisely because of its apparent naiveté:

The ‘naive’ definition of the set, given by Cantor, was: ‘A set is collection¹⁷ into a whole of definite and distinct objects of our intuition or of our thought. These objects are called the elements of a set.’ It is not despite, but because of, its undefined and undefinable terms, its circularities and its naive formulations (which rapidly led mathematicians to eliminate it and replace it by this or that group of axioms) that this alleged definition is fundamental. It displays precisely the undefinable, if not circular, character of the initial terms of set theory (and of all logic of mathematics) [...]. This essential characteristic [...] is hidden or covered over by the later elaborations. (IIS: 223)

There is a point here worth pondering. Formalizations of set theory are guided by our naive ideas about sets; axiomatic systems are, in this sense, artifices that appear a posteriori and, in some cases, in a somewhat ad hoc manner. Whatever ontological significance can later be attributed to, say, the Axiom of Separation, in the actual history of mathematical practice it was introduced on pragmatic grounds. It was introduced simply to prevent the appearance of known paradoxes, by a technique known for this reason as “monster barring”. Nevertheless, Badiou dismisses outright the objection that philosophy may be reading too much into these axioms: “I would like to insist on the fact that, it being set theory at stake, axiomatization is not an artifice of exposition, but an intrinsic necessity.” (BE: 43)

One can certainly imagine a rejoinder to Badiou. It may well be, one could argue, that axiomatization is necessary, but there are different axiomatizations and choosing one instead of another is a decision that is to some extent dependent on one’s political aims, aesthetic sensibility, or a sense of what is interesting or important. Castoriadis in fact comes close to formulating such an argument:

17. The article before “collection” may be deliberately absent: Castoriadis seems to be saying that a set is both a collection and an *act* of collecting. At least this is how I understand the operations of *legein*.

The fact that all the propositions of a given branch of mathematics can be reduced to a small number of axioms and deduced from them by means of a small number of schemata, substitution rules and deductive criteria, hides the equally—if not more—important fact that all the ‘axioms’ that could ‘freely’ be chosen are not of the same fecundity or interest, far from it. (IIS: 241)

This line of thought, however, is quickly abandoned in favor of arguing against formalization as such:

Formal mathematics is but the *caput mortuum* of mathematics as already constituted, it is not living mathematics in the making. If this were not the case, mathematics would be a simple *semeiotique*, that is to say, approximately the equivalent of what today is pompously called ‘semiotics’, and would exhibit the same depressing poverty as the latter. (IIS: 242)

Some historical evidence is alluded to in an endnote, partly as a counterfactual: “If Newton and Leibniz had had the misfortune to know the criteria of formalized mathematics, they would never have dared to publish their discoveries in differential calculus. Analysis was, logically, a seamy place for a century and a half, until the situation was cleaned up by Cauchy and Weierstrass.” (IIS: 398-9n15)

But avoiding formalization altogether is just as much of a political decision as the choice of a particular axiomatic system. We already see that it involves a certain stance toward semiotics, and it will also involve strong opinions about structuralism (hence also, at least implicitly, about structuralism’s entanglement with politics in the 1960s). To justify this decision Castoriadis offers a sequence of arguments, spread over a large number of pages in a slightly haphazard way. I present them in an order that is not faithful to the structure of the text but seems, at least to me, more logical. It is convenient to start with this observation:

it is of no matter, from the point of view we are interested in here, whether the Cantorian definition is criticized as naive and replaced by other more refined definitions in a more advanced formalism; all these formalizations presuppose signs posited and distinct and definite elements, united into a whole, the set of signs of the theory in question. Every set theory presupposes identity logic, and formalizable logic presupposes the ensemblizing of sign with which it operates. (IIS: 226)

As soon as there is a formal system, there is a set of symbols and therefore it is too late: the logic of sets is already in operation. But this is not quite sufficient for the sort of unlimited imaginative ontology Castoriadis prefers. Namely, one could concede the existence of *some* sets, for example some relatively small sets of symbols, and remain ontologically agnostic about others. In a way, this was the idea of Hilbert’s “formalist” program. Castoriadis targets this and related ideas in several places. There is, for example, an unsurprising reference to Gödel’s incompleteness theorem (IIS: 421), but this is not sufficient either. It only establishes that formal systems are incomplete, something Castoriadis agrees with from the outset. He must show that a “minimalist” ontology, which assumes only the existence of a set of symbols, is philosophically untenable. In his

framework set theory is not only a type of logic but also an ontology which is inseparable from logic.

What seems to be the main argument proceeds on two fronts simultaneously, against formalism and structuralism, both of which suppress the essential historicity of *legein*, and do it in a way that is described, rather dramatically, as a “murder of the object” (more precisely: the structuralist murder, as we saw earlier). The “scientific” synchronic view, according to Castoriadis, leads toward a linguistic relativism grounded in nothing but arbitrary convention, from which it can only recover by invoking a platonic fantasy of an atemporal space of universal place-holders of signs. In the case of structural linguistics this “space” of pure difference is known as *langue*; in the case of formalism, it is the “space” of infinitely repeatable content-free symbols (e.g., “stroke numerals”, or Pythagorean pebble- or dot-diagrams).

No less of an authority than Bourbaki, whom many regarded as a structuralist of sorts (though the collective’s philosophical views were somewhat hazy and deliberately obfuscated), provides Castoriadis with an opening:

the venerable author of the beautiful ‘Introduction’ to the *Elements of Mathematics* is somewhat lacking in rigor when he writes: ‘It goes without saying that the description of a formalized language is done in ordinary language, like the description of the rules of chess; we shall not enter into a discussion of the psychological or metaphysical problems raised by the question of the validity of the use of ordinary language in such circumstances [...]’. (IIS: 240)

Eager to avoid “metaphysical” questions—namely: philosophy, or a commitment to a particular philosophical option—Bourbaki casually reduces mathematics to arbitrary conventions of ordinary language. Mathematical formalism conceals this embarrassment by using artificial symbols that transcend the particularity of natural languages. But the rules for the correct manipulation of the allegedly universal symbols are described in ordinary language, and are thereby unmasked as mere conventions, “accepted” or “chosen forms”:

if mathematics were simply the ordered manipulation of signs [...], statements and demonstrations would be only arrangements of different order iterations of the ‘single’ sign ‘:’; and the ‘rules’ determining what is a ‘well-formed formula’ and a ‘demonstration’ would, in fact, be simply the ‘accepted’ or ‘chosen forms’ of the spatial arrangement of points [...]. (IIS: 194) [...] We know that this is a chimera, pursued for some time by some great mathematicians, but abandoned for 40 years [i.e., since Gödel], and which has now reappeared as the broken-down horse mounted by successive waves—ethnological, linguistic, psychoanalytic, semiotic—of Parisian fashion. (IIS: 396n34)

Assuming Castoriadis’s argument holds, we can pick up the thread we left off a while ago: logic does not simply presuppose the existence of a set of symbols. It is inextricably tied with the existence of sets “in the Cantorian sense”, and unleashes the full ontological power of *legein*:

identitary logic can be formulated if and only if there are, if there exist sets in the

Cantorian sense; *a fortiori*, then, it can be put to use only on this condition. One might think that ‘there are’, ‘there exist’ are here no more than strict references to the possibility of pure designation. But if this were the case, this designation would, precisely, remain pure. To say that identitary logic can be formulated and put to use therefore amounts to saying that there actually are sets, that sets *actually* exist. But at the same time, too, sets exist only in and through identitary logic, only in and through *legein*. In this sense, identitary logic, like *legein*, is equivalent to an ontological decision concerning what is and the manner in which it exists: what is is such that sets exist [...]. A decision that is at the same time the expression of a creation, of an ontological genesis: sets, these sets and the *eidos* of the set are henceforth posited-instituted and as such exist in a new region of being. (IIS: 227)

Before examining this critically it has to be said that on the level of an informal narrative the picture emerging from Castoriadis’s book is not an unattractive one. Mathematics involves imagination and creativity; the heuristic attribution of meaning to mathematical symbols draws on available imagery and in this sense it is part of the social-historical imaginary; conversely, it can transform the social-historical imaginary by, to paraphrase Wittgenstein, extending the domain of the imaginable. What made the discovery of irrational numbers “scandalous” to the Pythagoreans was precisely the fact that such objects exceeded the limits of the mathematical ontology to which they were religiously committed. Mathematics interlaces with an ontological framework, but it can participate in changing the framework by an “ontological decision”: from this point onward, irrational numbers exist, they are part of ontology and of the history of being *qua* being.

This is perhaps not so different, again on an entirely informal level, from the narrative emerging from Badiou’s work. He writes, for example: “Why must one always decide upon what exists? [...] Existence is precisely Being itself in as much as thought decides it.” And moreover: “Laturéamont’s praise of ‘severe mathematics’ (*mathématiques sévères*) is befitting. What is so severe is not so much its formalism and demonstrative entailment. Rather, it is the laying bare of a maxim of thought that can be formulated thus: it is when you decide upon what exists that you bind your thought to Being.”¹⁸

There are of course crucial differences, political and mathematical, some of which I already discussed. But there is another point of convergence which I feel I should mention. It has to do with a certain ethics: an ethics, one might say, of the infinite. Thus when Badiou writes in *Ethics* that we must *think* Man as something other than a being-for-death, other than a finite, mortal being, or that subjectivation is immortal—in the sense of being an infinite process—or, in the essay on Deleuze in the *Pocket Pantheon*, that “we must concede nothing to the hateful spirit of finitude”, it is difficult to ignore a number of related assertions by Castoriadis:

Why, in Kant and in Heidegger (and, in fact, in all of philosophy) is man a ‘finite being’? (Let us leave aside the strangeness of this expression, obviously meaningless—for man is not a number, and I do not know what finite means outside of mathematics and the mathematizable—and which can acquire meaning

18. *Briefings on Existence*, 55, 57.

only in reference to a theological phantasy and its translation into a philosophical thesis on God's infinity). Man is a 'finite being' not by reason of the 'banalities' represented by his mortality, his temporal anchoring, etc.; philosophically speaking, man is a 'finite being' [on the view to which Castoriadis here objects] *because he can create nothing*. (IIS: 199)

Like Badiou, who states that "invention and creation remain incalculable", Castoriadis is concerned with presenting a system that leaves room for imagination and creativity. His objection to traditional platonism, which presents the world as the "image of" some ahistorical space, is that it permits creativity only in the form of discovering what was already there. In fact this is his objection to much of philosophy. For example,

Heidegger and 'Marxists' meet curiously (in appearance) in the theme of 'production'—the sense of which, clearly, (*pro-ducere, hervor-bringen*, placing before, making come before) can only be, precisely, the one implied by Heideggerian ontology: the 'disclosure', the placing-before of what remained hidden but, of course, was already there. (IIS: 198)

If all production is disclosure of what was "already there", it would seem that the only genuinely creative activity is ontological. New sets appear in new regions of being through ontological decisions, which rely on the universally available operations of the social-historical imaginary but are beyond the reach of any formal justification. It is instructive to turn to Badiou, for a moment. In his system, existence (and truth-) claims are limited both by a strict formalization (including the expectation of logical consistency) and by the fact that ontological decisions are not entirely arbitrary—they occur in response to an event. Nevertheless, even in the more rigorous setting of Badiou's system, decisions are being made about the undecidable. Thus the problem arises of two subjects making (or being made by) mutually contradictory ontological claims in response to the same event; or simply naming the same event in different and politically contentious ways.

Badiou has been made aware of the difficulty: "Lyotard said that I was an absolute decisionist, a sort of new Carl Schmitt."¹⁹ The allegation is unpleasant enough, but for Castoriadis the underlying problem appears to be worse. Ontological decisions seem to be arbitrary creative acts of the social-historical imaginary, "which is creation *ex nihilo*": they do not seem to be limited in any way whatsoever. Hence different societies could, in principle, have completely disjoint ideas about being, even if they all operate with the same ensemblist logic of *legein*.

Castoriadis is thus forced to look for an anchor in something less abstract than set theory; something like nature. Presumably most of us can agree about that: there is nature, or something like it, common to diverse experiences of the world. But Castoriadis cannot quite call it "nature"—that would send him back toward the more vulgar forms of dialectical materialism and the kinds of Marxism of which his theory is supposed to be a revolutionary surpassing. He therefore introduces a surrogate of nature, namely "the first natural stratum" on which all societies "lean". The first natural stratum is the

19. *Infinite Thought*, 129.

social-historical imaginary's construction of nature. It sort of does not exist, but it has to be more than imaginary, so it kind of exists:

It is almost non-existent when we consider the tenor of the instituted imaginary significations as such. But at the same time it cannot be eliminated, not as the (trivial) physical and biological condition for the existence of society, but more especially as the logical support, the point of anchorage of the actual ensemblization [...], fixing the terms of marking without which imaginary significations would have no point of reference. (IIS: 230-1)

This is stated in a discussion of the social status of children, a case in which “the leaning of the institution on the natural stratum [...] appears and actually is vague and remote”. But in fact the passage cited above applies willy-nilly to Castoriadis's entire system, and amounts to an admission. Something nature-like has to exist, it has to be ontologically available everywhere, so that all societies can “lean” on it. Only then can the social-historical imaginary's positing of sets retain even a semblance of universality—a minimum consensus on Being, the condition of possibility of the idea that, in some sense at least, and despite our innumerable differences, we all share (parts of) the same world, even if it is constructed differently. That is indeed both a philosophical and a political problem.²⁰

Part of the point of using set theory is to overcome the orthodox Marxist understanding of “the science of history”, which relied on metaphors drawn from Newtonian physics and classical mechanics; its world is, “in the last instance”, deterministic. There is no such determinism in an ontology of sets: sets cannot be said to *cause* anything about other sets. With a very different political motive, Cantor himself attempted a set-theoretic argument against determinism; hence his celebrated assertion that “the essence of mathematics is in its freedom”. But Castoriadis is not a theologian and needs a firm non-theological reference point in order to avoid relativism in the form of completely arbitrary ontological decisions. And his attempt to find a nature-like anchoring point of reference puts him on a slippery slope toward yet another version of the “dialectic of nature”.

It is in the direction of this problem that Badiou—the fictional Badiou of this article, at any rate—sends a fatal theorem. He knows just how to define “nature” in set-theoretic terms so as to be able to deduce, within a certain formalization of set theory, that “nature does not exist”.²¹ Without nature, or the first natural stratum of socially-imagined naturally-anchored set-objects, Castoriadis's system, despite its frequently lucid references to mathematics, becomes a theory of the “magma of social significations”, or even a “magma of magmas”. What remains of nature appears in the end to function as a Lacanian *point de capiton*, something that arrests, anchors, or “sutures” the magmatic flow of imaginary constructs, as described by Žižek:

What creates and sustains the identity of a given ideological field beyond all

20. In *The Meaning of Sarkozy*, Badiou takes the assertion that there is only one world as a motivation for political action.

21. See Meditation Twelve of *Being and Event*, esp. page 140.

possible variations of its positive content? “Hegemony and Socialist Strategy” delineates what is probably the definitive answer to this crucial question of the theory of ideology: the multitude of ‘floating signifiers’, of proto-ideological elements, is structured into a unified field through the intervention of a certain ‘nodal point’ (the Lacanian *point de capiton*) which ‘quilts’ them, stops their sliding and fixes their meaning.²²

This is not the place to expound on how Badiou resolves analogous problems, or whether he resolves them at all. It must be said, nevertheless, that one of the issues raised by Castoriadis, though only implicitly and in passing—the issue of the *choice* of axiomatic system—posed a serious difficulty for Badiou of *Being and Event*. It is for this reason, among other reasons, that in his later work he abandons axiomatic set theory and introduces a new mathematical instrument, topos theory, that allows a plurality of “local” set theories and a plurality of logics.

In *Being and Event* the Zermelo-Fraenkel axiomatization is chosen because it is generally taken as standard (though in truth the majority of practicing mathematicians hardly care), but also because it does not permit collections that are in a technical sense too large to be sets, such as the class of all sets. This in turn allows Badiou to claim for set theory the status of a “materialist” ontology that does not admit an absolute Being. But this is a feature of a particular axiomatic system. In Quine’s axiomatization of set theory, which is also “materialist” in the sense that it limits the ontic powers of language, there *is* a set of all sets. Quine’s system does not allow all formulas to induce existence, but it does allow a formula that defines a universal set.

There are different theories that are “materialist” in Badiou’s sense. However, the resulting ontologies are not the same, and choosing one over another is not only an act of “binding your thought to Being”, but a political (as well as aesthetical) choice. This in turn places mathematics, that is ontology itself, under the jurisdiction of the political condition of philosophy, or even the political condition of a particular philosopher.

For example, in the framework of the von Neumann-Bernays-Gödel theory Badiou’s clever but bizarre definition of “nature” would not have the same effect as it does in *Being and Event*: the collection of ordinal numbers, which Badiou calls “nature”, is not a set but it *is* part of the mathematical ontology of this axiomatic system. Thus the question of the choice of formalization, raised almost in passing by Castoriadis, seems to haunt *Being and Event* no less than the absence of formalization haunts Castoriadis’s *legein*.

It is of course easy to imagine that in reality Badiou did not bother reading *The Imaginary Institution of Society*, or not in as much detail as I think the book deserves; and that Castoriadis would regard Badiou’s imperative of formalization and reinvention of platonism as misguided, worthy perhaps of a characteristically acerbic endnote. Yet it is oddly compelling, to a distant observer such as myself, to imagine a world in which they would have had more to say to one another.

22. Slavoj Žižek, *The Sublime Object of Ideology* (London, Verso, 2009), 82.