THE LIMITS OF THE SUBJECT IN BADIOU'S *BEING AND EVENT*

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ABSTRACT: This essay is an examination of the limits of the model of the subject that Badiou establishes in *Being and Event*. This will concentrate on both *Being and Event*, and the later ethical developments introduced in *Ethics: An Essay on the Understanding of Evil*. My aim will be to show that there is a possible subjective figure, based on the independence of the Axiom of Choice, which remains unexamined in both these works. The introduction of this new subjective figure not only complicates Badiou's ethical categories of Good and Evil, but it also raises questions about the nature of the subject in general in his philosophy.

KEYWORDS: Badiou; Axiom of Choice; Subject; Individual; Non-constructible Sets; Temporality

The figure of the subject in Badiou's *Being and Event*[1] is key to understanding the link between his revival of a systematic ontology, in the form of set theoretical mathematics, and his wider philosophical and ethical concerns. Through a critical examination of the subject, as it appears in *Being and Event*, and an evaluation of the categories of subjective Good and Evil, developed in his book *Ethics: an Essay on the Understanding of Evil*[2], I hope to probe the limits of this subjective model and to propose a new subjective figure that appears possible, but unexamined, in either of these works.

My analysis will focus on two main points: first, Badiou's use of the Axiom of Choice, as a key factor in his philosophy that allows for the possibility of a subject, and, second, his selective use of set theoretical forcing, which concentrates mainly on the independence of the Continuum Hypothesis.

Badiou's ethics is based on the capacity of individuals to distinguish themselves from their *finite* animal nature and to become *immortal*; to become immortal is to become a subject (E 12, 132). What constitutes this singular ability, our rationality, is the use of mathematics (E 132). Specifically it is the Axiom of Choice that elevates the human animal to the level of a *potential* subject. This axiom expresses an individual's freedom, a

1. Alain Badiou, *Being and Event*, trans. Oliver Feltham, London, Continuum, 2005 (henceforth BE).
2. Alain Badiou, *Ethics: an Essay on the Understanding of Evil*, trans. Peter Hallward, London, Verso, 2001 (henceforth E).

freedom equivalent to the affirmation of pure chance.[3] It is this capacity that allows an individual to affirm its chance encounter with an event; the moment of this affirmation is called *intervention* and marks the birth of a subject (BE Meditations 20 and 22).

The importance of the Axiom of Choice is clear; it provides the connection between the individual, the event and the subject. It defines the individual and provides the *condition* under which subjectivity is possible.

Badiou's appeal to Paul Cohen's theory of forcing is predominately directed toward his proof of the independence of Georg Cantor's Continuum Hypothesis. But in Cohen's book, *Set Theory and the Continuum Hypothesis*, the method of forcing is used equally to prove the independence of the Axiom of Choice.[4] For Badiou, the Continuum Hypothesis is a restrictive theorem of ontology; it confines ontology to the merely constructible and neuters the individual by reducing the power of the Axiom of Choice (BE Meditations 28 and 9). Under such a restriction the Axiom of Choice loses its independence as an axiom and becomes a theorem, a mere consequence of the system (BE 305-7). Cohen's theory of forcing is important as it shows that it is possible to construct a model of set theory in which the Continuum Hypothesis fails, thus liberating us from its restrictive bonds. In the process it not only reinstates the full power of the Axiom of Choice, the freedom of the individual, but also, through the use of this axiom, a *subject* emerges.

The mathematical theory of forcing, as it is applied to the Continuum Hypothesis, provides Badiou with the paradigmatic model for the subjective response to an event. The subjective process *emancipates* the individual, through a *correct* use of their freedom in the face of an event, from some restrictive condition of their situation. This production of a truth introduces true novelty that expands, or extends, the subject's situation. This forms the basis of Badiou's theory of ethics. Subjective endeavour, forcing the truth of an event, forms the *positive* concept of the Good and 'It is from our positive capability for Good… that we are to identify Evil' (E 16). The range of types of Evil can be identified with false, abortive or totalizing activities that try to subvert a truth procedure, the Good being the practice of the *virtues* of discernment, courage and moderation (E 91). The subject remains faithful to the event and its consequences.

The clarity and decisive character of Badiou's ethics is refreshing, but is it the case that the subject is always *intrinsically* good? What would happen if we examined the consequences of a valid subjective process, based on the mathematical model of forcing, which instead of liberating an individual, in the process of their subjective action, condemned them? I think the independence of the Axiom of Choice provides such an occasion. What would be the consequences of forcing a situation in which the Axiom of Choice fails, in which the freedom of the individual is denied and the competence of the subject questioned?

To be in a position to evaluate the nature of this possible subject it will be necessary to fully understand Badiou's move to equate ontology and mathematics, and to recog-

3. Alain Badiou, '*On Subtraction*' in *Theoretical Writings*, ed. and trans. Ray Brassier and Alberto Toscano, London, Continuum, 2004, p. 113.
4. Paul Cohen, *Set Theory and the Continuum Hypothesis*, New York, W. A. Benjamin, 1966, pp. 136-142.

nize that his theory of the event is more than a reduction of philosophy to mathematics.

It will therefore be necessary to examine two main areas: first, the reasons why Badiou equates ontology and mathematics, focusing on the critical distinctions this allows him to make, and, second, the importance of the Axiom of Choice for the formation of the subject, and the specific relation between the subject and the event. Special attention will be given to those concepts that separate the theory of the subject from its ontological existence, namely the matheme of the event and the concepts of History and temporality. Finally I will consider Badiou's ethics, and the types of subjectivity associated with Good and Evil and conclude with an analysis of the position and character of a subject based on the procedure of forcing a situation in which the Axiom of Choice fails.

I. ONTOLOGY, SET THEORY AND THE SPACE OF THE SUBJECT

Badiou's philosophical claim that mathematics *is* ontology forms the central thesis of *Being and Event* (BE 4). One of the main figures that motivate this approach is Heidegger and his critique of Western metaphysics. Like Heidegger, Badiou believes that philosophy can only be revitalized through a new examination of the ontological question, but he does not agree with his later retreat into poetics (BE 2, 9-10).

Badiou sees Heidegger's problem in his refusal to give any legitimacy to systematic ontology. This refusal is based on the belief that systematic ontology always begins with the move of forcing an identity between *being* and the *one*, or oneness.[5] This identity causes being to split into separate essential and existential parts; the history of metaphysics then exhausts itself in the impossible task of reconciling and rejoining these two aspects.[6] The solution to this problem is to view metaphysics as something that must be abandoned, its positive role can only be to make the question of being ever more poignant through the distress that it causes: this distress is heard as the cry elicited by the violation of being by metaphysics. The truth of being can only be understood as the simple letting be of being, exemplified by poetic thought that refrains from all analysis. Here being is thought of as a simple presencing, where the two aspects of the essential and the existential belong together in an undifferentiated shining forth, prior to any separation.[7]

Badiou's response to Heidegger is twofold: to separate philosophy from ontology and to propose a systematic ontology not based on the one. This last point gives rise to what he calls his ontological wager: 'the one *is not*' (BE 23). There is no pure presentation of being, not even the poetic active *presencing* of Heidegger, instead being is radically subtracted from all presentation (BE 10). The problem with the history of philosophy

5. Martin Heidegger, *Contributions to Philosophy: from Enowning*, trans. Parvis Emad and Kenneth Maly, Bloomington, Indiana University Press, 1999, p. 146 §110:4.
6. Heidegger, *Contributions to Philosophy*, pp. 145-146 §110:2.
7. Heidegger, *Contributions to Philosophy*, pp. 145-146 §110:2.

has not been its attempt to present being in a *consistent* and *systematic* way, but its attempt to present being as a *one*. For Badiou, if being is not a one, then it can only be thought of as a pure multiple: being *is*, but it is not one, therefore it must be multiple. Here we have the two key conditions for ontology: being *is* multiple and the one *is not*. Ontology must be the consistent presentation of the pure multiple of being; the problem is that consistent presentation involves the one, or oneness (BE 23-4). Badiou avoids conceding a point of being to the one by conceiving it as a pure operation, the operation of the count-as-one (BE 24). The one, for Badiou, must remain a process, therefore the one as this operation of the presentation of the count-as-one is never itself presented; it is only the structure of presentation. It is *how* the multiple is presented, not *what* the multiple is. Hence oneness is presented as the result of the operation of the count-as-one on the pure multiplicity of being, as Badiou states: 'What will have been counted as one, on the basis of not having been one, turns out to be multiple' (BE 24). This move enables Badiou to make a decisive distinction, that between *consistent* and *inconsistent* multiplicity (BE 25). These distinctions apply to the pure multiplicity of being as it is split apart by the operation of the count-for-one, into a retroactively designated prior *inconsistency* and a *consistent* result as a presented one.

Before examining in some detail the appeal that Badiou makes to set theoretical mathematics in order to realize this ontology, it is worth considering what he hopes to achieve by adopting such an approach. What Badiou is essentially trying to achieve is to move philosophy beyond its obsession with foundations, origins and beginnings. Philosophy should not only give up its search for foundations, but also its post-modern lament on the impossibility of such origins. For Badiou, the creation of novelty, in the form of a truth produced by subjective endeavour, does not find its source in the impossibility of presenting being, an impossibility whose trace resides in all presentation, but in fidelity to an event (BE 27). The subject affirms that something has happened and is prepared to bare the consequences, whether the event actually occurred may be undecidable but the situation provides the subject with the necessary material to not only distinguish different events, but also recognize the problem posed by the event as different from the problem of foundation.

Badiou's aim is to establish two fundamentally different concepts of non-relation that he feels have been confused in philosophy. The first is the type of non-relation described above: there must be no relation between being and the one. This is the unilateral subtraction of being from presentation: the inconsistent multiple is never presented, only ever a consistent presentation of it. This type of non-relation is a *no*-relation, ontology is a situation that presents a structure, but being has *no* structure (BE 26-7). Relations, or functions, are always consistent ontological presentations, but they do not always share the same degree of determination (BE Appendix 2).

The second type of non-relation is more of a non-determinate relation. Consistent multiples within Badiou's ontology can usually be subject to two different kinds of presentation, an *extensive* presentation, associated with a multiplicity's *cardinal* magnitude, and an *intensive* presentation, associated with its *ordinal* order. These two types of presentation form two separate number systems: the cardinal and ordinal numbers. At the finite level these two systems coincide and behave identically, but at the infinite level the two systems diverge and their relation to each other becomes indeterminate. A space is opened up at the infinite level whereby a multiplicity of possible relations can be maintained between the two systems. More importantly, for Badiou, is the possibility under certain restricted situations for multiples to exist which have no intensive presentation, only an extensive one: these multiples are called non-constructible. Such non-constructible multiples provide the material that a subject requires in order to transform a situation. What is important about this type of multiple is that it is a form of *unordered* consistent presentation. Consistent presentation is not dependent on order; it is not constrained to what can be constructed, but can encompass the minimal structure of unordered, or disordered, multiplicity. This lack of structured order is not to be confused with a lack of consistency: the disordered is *not* inconsistent.

For Badiou, ontology must be able to make this distinction between indeterminacy, in terms of disorder, and inconsistency. His recourse to set theory must therefore achieve three things: first, it must establish that an ontology based on the pure multiple is possible; second, that there is within this system of ontology indeterminate, or indiscernible, material and, third, that this material can be accessed and utilized by a subject. The event itself, which motivates a subject, is always outside and excluded from ontology (BE 189-90). The remainder of this first section will concentrate on the first two points: axiomatic set theory as a possible ontology of the pure multiple, and the significance of the infinite within set theory for introducing the concept of the non-constructible set and the indiscernible.

a) Set Theoretical Foundations

Badiou's philosophy stands or falls on whether set theory actually provides an ontology of the pure multiple that avoids the pitfalls of the one. Only after this possible use of set theory has been accepted can we begin to look at how Badiou uses it in his theory of the subject. The first few meditations of *Being and Event*, which introduce set theory, are motivated only by the desire to demonstrate that such an ontology is possible.

It is not clear how set theory can provide a theory of the pure multiple, which avoids attributing being to the one. Even if we accept that the count-for-one, as an operation, avoids presenting being as a one, and only attributes oneness to the structure of presentation, an idea that is not without its critics, this still leaves us with an empty theory.[8] Badiou thinks that the formal axiom system of Zermelo-Fraenkel set theory (ZF) avoids making what is presented in the operation of the count-for-one into *a* being by excluding any formal definition of a set (BE 30). What a set *is* cannot be defined; being is never attributed to the *concept* of a set. A set might be thought of as the collection of its members into a one, they are counted as one, but the set's members are again sets. A set is the

8. Jean-Toussaint Desanti, 'Some Remarks on the Intrinsic Ontology of Alain Badiou', in Peter Hallward (ed.), *Think Again: Alain Badiou and the Future of Philosophy*, London, Continuum Press, 2004.

presentation of pure multiplicity; the members of a set are multiples of multiples of multiples, endlessly. There can be no formal definition of what a set is, as our understanding of it is dependent on us already knowing what a set is, the alternative, discussed below, is to designate atomic entities that are not themselves sets. The oneness inherent in the presentation of a set is due to the operation of presentation; it is not dependent on any inherent oneness in what is being presented.

Subsequently the majority of the axioms of ZF dictate rules for the formal manipulation of sets, but they do not entail the actual existence of any set (BE 62). If an axiom cannot be given that either discerns or generates sets then, to prevent the system from being empty, it is necessary for axioms to explicitly state the existence of certain sets.

Here the danger of reintroducing the one can occur, depending on what type of sets are claimed to exist. There are many different ways of introducing sets axiomatically, but they do not all provide a pure theory of the multiple. It is not sufficient to simply use a formal axiomatic system, it is also important that the right axioms are chosen. There are many theories of set theory that introduce atomic *individuals* at the axiomatic level, which, in Badiou's eyes, would clearly constitute the presentation of being as a one.[9] The axioms that do not conform to simple rules of manipulation are the two explicitly existential axioms of the Empty Set and Infinity (BE 62). The Axiom of the Empty Set, finally, allows Badiou to claim that set theory is a theory of the pure multiple. In order to understand the significance of this axiom it will be necessary to introduce some set theoretical terminology.

Badiou's initial introduction of the concept of the pure *presented* multiple, as the result of the operation of the count-as-one, is very close to Georg Cantor's original naïve description of a set: 'By a set we are to understand any collection into a whole M of definite and separate objects *m*'.[10] Such a set M is written: M = $\{m\}$, or if M has more than one element, M = $\{m_1, m_2, m_3, \ldots m_n\}$. A set is therefore a collection of separate elements, which are said to *belong* to a set. This relation of belonging is the fundamental non-logical relation that structures all sets, and is written '∈'. In the set M above, for example, all the elements that appear within the brackets belong to M: $m_1 \in M$, $m_2 \in M$, and so on. For Badiou, the set is the consistent presentation of its elements. The term element can be somewhat misleading, as it seems to suggest that the elements themselves are ones, thus introducing oneness into set theory. Badiou avoids calling these terms elements and prefers to call them presented terms. I will continue to call them elements as this is the name that most commonly appears in texts on set theory. The construction of the elements of sets will make it clear that they are not atomic individuals, but rather pure multiples, which are each multiple in their own right.

The initial set, asserted to exist axiomatically, cannot have any members; nothing can belong to it. If it did, the set's members could legitimately be held to be atomic individuals. This would guarantee that the one *is*, contradicting the wager that the one *is not*.

9. Michael Potter, *Set Theory and its Philosophy*, Oxford, Oxford University Press, 2004, pp. 72-75, 291.
10. Georg Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*, trans. Philip Jourdain, New York, Dover, 1915, p. 85.

Therefore, to begin with, the only set that can be asserted to exist, without contradicting the above wager, is an empty set. Unsurprisingly, the Axiom of the Empty Set asserts that just such a set exists. Badiou's technical formulation of this axiom is:

$$(\exists\beta)[\neg(\exists\alpha)(\alpha\in\beta)]$$

This reads 'there exists a β such that there does not exist any α which belongs to it' (BE 68). The set β is void, or empty. In his formulation Badiou chooses to use the existential quantifier, $\exists$, 'there exists', twice rather than the more usual use of the universal quantifier, $\forall$, 'for all'. The more usual formulation of this axiom is:

$$(\exists\beta)(\forall\alpha)\neg(\alpha\in\beta)$$ [11]

This would read: there exists a set β such that, for all α, no α belongs to β. The double existential form is important for Badiou: there *exists* β such that there *does not exist* α. There is the *presentation* of something that is *not* presented, for Badiou this is pure inconsistent multiplicity (BE 67).

With this axiom, the final requirements of a theory of the pure multiple, a form of consistent presentation without a one, is achieved. The metaontological significance of this axiom is that 'the unpresentable is presented, as a subtractive term of the presentation of presentation' (BE 67). As Badiou states: 'If there cannot be *a* presentation *of* being because being occurs in every presentation—and this is why it does not present *itself*—then there is one solution left for us: that the ontological situation be *the presentation of presentation*' (BE 27). The Axiom of the Empty Set guarantees the existence of at least one set, from which other sets can then be generated, but this set presents nothing more than presentation itself. The empty set, written $\varnothing$, can be thought of as simply an empty pair of brackets: $\varnothing = \{\}$. If a set is the formal operation of presenting its elements, then if a set has no elements all it presents is this formal operation itself: the empty set, $\varnothing$, presents nothing but presentation itself.

This *consistent* presentation is often assumed as paradoxical, or a sleight of hand: the assertion that $\varnothing$ exists means that the theory is not empty, only that the *content* of this theory *is* empty. What is being presented here is only the 'how', of how being can be presented: the operation of the count-as-one. The content of mathematical set theory is empty, although there is a great richness to the structures of presentation. The empty set in conjunction with the other ZF axioms can be used to generate an indefinite number of other sets, all of which ultimately present nothing. Therefore the theory is not empty, it is populated by the variety of empty structures of presentation, but it is still, finally, empty.

Here we can see how the two alternative phrasings 'the one *is not*' and 'being *is* multiple' are both satisfied by this axiom. Every result of a count-as-one, a set, is formed from the empty set, so that although the presentation is not empty, there is a presentation of structure, *nothing*, that is no *being*, is presented: the one *is not*. Being is therefore subtracted from all presentation of it as a one, the empty set perfectly expresses this by presenting nothing, no one, and if being is not one then it is multiple.

11. Mary Tiles, *The Philosophy of Set Theory: An Historical Introduction to Cantor's Paradise*, Oxford, Basil Blackwell, 1989, p. 121.

The final point to be made on this is that the empty set's uniqueness means that it acts as a proper name, the proper name of being. The empty set, $\varnothing$, is not the presentation of being itself, but only its proper name. The uniqueness of $\varnothing$ is immediate as *nothing* differentiates it; the uniqueness of the empty set is based on its in-difference (BE 68). The empty set, or void set as Badiou often calls it, is in-different *not* indiscernible. It is not that we cannot discern what is presented in the empty set, but rather that there is *nothing* to discern. This point is of vital importance when indiscernible sets are introduced as being central to a theory of the subject.

To conclude this section, set theory is based not on a general definition of a set, but on the assertion that a particular set does exist. The empty set, $\varnothing$, makes it possible for set theory to be an ontology of the pure multiple.

b) The Infinite as the Space of Novelty

Having established that such a form of ontology is *possible*, it is now necessary to show that it is not sterile. The space opened by set theory must not be foreclosed against novelty. The fear is that set theory will present such a formal system that it will be structurally determined and closed. Although this is true at the finite level, at the infinite level there is no absolute structure. For Badiou, the notion of the infinite does not go hand in hand with the themes of transcendence and totalization, but it is instead what makes the indeterminate and the undecidable possible. In this section I will explore how the concept of the infinite frees ontology from any single structure, and allows for the appearance of the indiscernible, or non-constructible set.

In order to make these aims clear it will be necessary to introduce more of the technical terminology of set theory of *Being and Event*.

Cantor's initial aim with his theory of sets was to introduce the most abstract mathematical objects possible: at base they should be pure multiples abstracted from both their *content* and their *order* of appearance.[12] Free from these two *intrinsic* qualities a set was presented as a pure *extrinsic* multiple. This idea remains in modern ZF set theory in the form of the Axiom of Extension, which defines the identity of a set solely in terms of its elements. A set is nothing more than the collection of the elements that it brings together, regardless of how these elements have been collected or arranged. The axiom states:

$$(\forall\gamma)\ [(\gamma\in\alpha) \leftrightarrow (\gamma\in\beta)] \rightarrow (\alpha=\beta)$$

This reads: a set α is the same as a set β if, and only if, every element of α is also an element of β, and vice versa. This extensional, or combinatorial, concept of a set is vital for Badiou; a set is a pure multiple defined by nothing more than the multiples that it presents.

Cantor called this abstract extensional presentation of a set its *power* or its *cardinal* number, but it is also possible to think of a set in terms of its *intrinsic order*, thus defining the sets *ordinal* type. If a set is *well ordered*, the ordinal type of the set becomes its ordinal

12. Cantor, *Contributions to the Founding of the Theory of Transfinite Numbers*, p. 86.

number. A set is partially ordered if each element can be thought to 'have a place' relative to the other elements. For every m_1 and m_2 belonging to a set M, and $m_1 \neq m_2$, it must be the case that either $m_1 < m_2$ or $m_2 < m_1$. This equates with our general understanding of the natural, rational and, even, the real numbers. Well ordering is a slightly more strict form of order, which restricts well ordering to the type of discrete order found only in the natural numbers, each number always has a direct successor with no number appearing between the original number and its successor.

Two sets α and β have the same cardinal number if there is a one-to-one relation between them; each element of α maps onto a unique element of β and vice versa. Two sets α and β have the same ordinal number if a similar one-to-one relation exists, but the relation must also preserve the well ordering of the sets.

It is this distinction between a set's cardinal and ordinal character, and the relation between these two relations, that lies at the heart of both Cantor's life long obsession with the continuum hypothesis, and Badiou's interest in set theory and the infinite.

The difference between cardinal and ordinal numbers is simple to understand, but the significance of this distinction does not become obvious until infinite sets are considered. Cardinality measures the magnitude of a set, while ordinality is a measure of degree, based on order. Take for example the set $\alpha = \{1, 2, 3, 4\}$, this set has a cardinal power of four and an ordinal degree of four. It has a cardinal power of four, as it clearly has four elements. It has an ordinal degree of four, as the highest ranked element, according to its ordering, is four. If a set has a clear order then we need only look for its highest ranked element in order to know its ordinal number.

At the finite level every set can be well ordered, also this ordering is unique: you cannot change the ordinal value of a finite set by rearranging its elements. Every finite set can only be associated with one ordinal number. This ordinal number is also identical to its cardinal power; in the above example the set α had both the ordinal and cardinal number four.

The concept of an infinite ordinal can only be reached through an extension of the method that generates finite ordinals. This is the seemingly simple notion of adding one. Badiou's approach to the construction of the ordinals begins with his distinction between belonging and inclusion. Badiou claims that this distinction provides the source of the originality of *Being and Event* (BE 81).

Given a set $\alpha = \{a, b, c, d\}$, the elements that belong to it are: a, b, c and d. But what about sets that share coincident elements, such as $\beta = \{a, b\}$ for example? Such a set is said to be *included* in α, or to be a *subset* of α, and is written: $\beta \subset \alpha$. If all the elements of a set β are also elements of α, then β is a subset of α. The Power Set Axiom then states that if a set α exists then so does the set of all α's subsets. Taking the example $\gamma = \{a, b, c\}$, the power set of α is: $\wp(\alpha) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \varnothing\}$. The new set, $\wp(\alpha)$, has eight, or 2^3, elements. Perhaps the only two surprising inclusions are the empty set and the set γ itself. Given the definition of a subset above, their inclusion becomes clear. Although the original set cannot belong to itself, on pain of paradox and inconsistency, it can include itself as it obviously shares all its elements.[13] The empty set, $\varnothing$, has the unique property of being universally included in all sets; there is *no* element belonging to $\varnothing$, which is not also an element of any other set, as $\varnothing$ has no elements.

Before continuing, it is worth noting how important the Power Set Axiom is for Badiou. If sets *present* their elements, they *represent* their subsets. The full representation of a set is equivalent to its power set, and Badiou calls this the State of a situation (BE 95). The State represents the situation, and it will be in the minimal relation between an infinite set/situation and its power set/State that novelty will be possible.

Badiou's set theoretical universe is, to begin with, sparse; only the empty set exists. The first new set he produces is $\wp(\varnothing) = \{\varnothing\}$, a set with one element, a singleton (BE 91). This is not too surprising either, if the general rule is that the number of elements of a power set are 2^n, where n is the original number of elements, if $n = 0$ then $2^0 = 1$. From this Badiou derives the rule that given any set α, then its singleton, $\{\alpha\}$, also exists (BE 91).[14]

We are now in a position to consider the construction of the finite ordinals. The void, or empty set $\varnothing$ can be considered as the first natural ordinal 0, with its singleton $\{\varnothing\}$ corresponding to the ordinal 1. The successor of these two ordinals is the union of these two: $\varnothing \cup \{\varnothing\} = \{\varnothing, \{\varnothing\}\}$, the ordinal 2. The process of succession is to form the unity between the current ordinal and the singleton of this current ordinal. The construction of the ordinal 3 is accomplished as follows: the union of $\{\varnothing, \{\varnothing\}\}$ with its singleton $\{\{\varnothing, \{\varnothing\}\}\}$: $\{\varnothing, \{\varnothing\}\} \cup \{\{\varnothing, \{\varnothing\}\}\} = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$. In general if a is an ordinal the successor of a is $a \cup \{a\}$, this is equivalent to the idea of adding one. The interesting feature of this construction of the ordinals is that all the previous stages of the construction appear within the current level as elements. Every element of an ordinal is itself an ordinal, it is this feature of nesting and homogeneity that qualifies as a set as *transitive*:

$$\forall\alpha\forall\beta\ (\alpha\in\gamma\ \&\ \beta\in\alpha) \rightarrow \beta\in\gamma$$

This reads, if α belongs to γ and β belongs to α, then β belongs to γ.[15] Badiou calls such transitive sets *normal* and recognizes them as the hallmark of natural situations (BE 132-4). Every ordinal number is a *transitive* set, well ordered by the relation of belonging.

This method can then be used to generate any finite number of ordinals. But it cannot be used to create an infinite set, one greater than all the finite ordinals. The nature of ordinal numbers means that an ordinal greater than all the finite ordinals would include all these ordinals as elements. It would be the set of all ordinals that could be produced

13. Such paradoxes include Russell's paradox etc.

14. Badiou suggests this as an application of the Axiom of Replacement, where the element of the singleton $\{\varnothing\}$ is replaced by an arbitrary set α, to form the singleton $\{\alpha\}$. It also follows from the Power Set Axiom, where the singleton can be thought of as the power set of α, minus everything that is not α. For example, if $\alpha = \{a, b\}$, then $\wp(\alpha) = \{\{a\}, \{b\}, \{a, b\}, \varnothing\}$, if we remove the subsets $\{a\}$, $\{b\}$ and $\varnothing$ we are left with $\{\{a, b\}\} = \{\alpha\}$, the singleton of α.

15. Tiles, *The Philosophy of Set Theory*, p. 134.

using the method of simple succession, the limit of this productive procedure. This *limit* ordinal is called ω, and can only be introduced via a second existential declaration (BE 156). The Axiom of Infinity states: there exists a set ω, such that for any finite ordinal a, both a and the successor of a, $a\cup\{a\}$, belong to ω. Although there is a first infinite ordinal, there is no last finite ordinal (BE 159).

There are now two types of ordinal numbers, the finite ordinals *produced* by means of succession, and the infinite ordinal ω, stated to exist as the limit of the process of succession. Hence we have successor and limit ordinals. It is now possible to examine the profound differences between an ordinal and cardinal conception of number.

Ordinal succession can be reintroduced, without modification, at the infinite level. There is the *next* ordinal after ω, which is $\omega\cup\{\omega\}$, or $\omega+1$. Again, an infinite number of new ordinal numbers can be created, their structure being defined by the number of times the above two modes of generation are used. For example, the set of all the even numbers followed by the set of all the odd numbers, $\{2, 4, 6, 8\ldots ; 1, 3, 5, 9\ldots\}$, uses in its *intrinsic* structure the rule of the limit of the process of succession twice, its ordinal number is therefore $\omega\cdot 2>\omega$. Cardinality, on the other hand, takes no notice of the intrinsic ordering of a set and measures the pure magnitude in terms of the number of elements. The cardinality of one set is said to be equal to that of another if a simple one-to-one relation is possible between them. This is trivial for the above example, 1 would map to 1, and 2 to 2 and so one. Therefore the cardinality of the set of even numbers followed by the set of odd numbers is equivalent to the cardinality of the set of natural numbers. It is no longer the case that every ordinal set can be associated with a unique cardinal number. An infinite number of infinite ordinals share the same cardinality, all of them equivalent to ω. The cardinal number associated with ω is $\aleph_0$, aleph null, and all ordinal sets using the first two methods of construction share the same cardinality.[16]

After the rather benign and simple relation between cardinal and ordinal numbers at the finite level, their divergence at the infinite level is quite fascinating. The question now arises: what is the relationship between an ordinal set's intrinsic ordinal number and its extensive cardinality?

In order to make the ordinal number system a closed and coherent system Cantor added a third rule of ordinal generation, to add to the two rules of succession and taking the limit of a succession.[17] The first rule generates all the finite numbers, and these constitute the first class of ordinal numbers (I), the combination of the first rule with the second produces all the infinite ordinals with a cardinality of $\aleph_0$, and constitutes the second class of ordinal numbers (II). The third rule of generation, called the Principle of Limitation, states that a new class of ordinal numbers (III) can be generated by taking the aggregate of all the ordinals that can be produced using the first two rules. This new ordinal, ω_1, has a cardinality that exceeds $\aleph_0$, and is thought of as the next cardinal

16. This leads to the familiar proofs that the set of even numbers is equinumerous with the set of odd numbers, and that the natural numbers are equinumerous with the rationals.
17. Potter, *The Philosophy of Set Theory*, p. 106.

after $\aleph_0$ called $\aleph_1$.[18] An important feature of the ordinal ω_1 is that, because it cannot be put into a one-to-one correspondence with the denumerable natural numbers, it is non-denumerable or uncountable.

This method can be used to generate an indefinite series of ordinal number classes; the ordinals of each class have the same cardinality as the aggregate of all the ordinals in the class below. The first ordinal of each class is known as a limit ordinal and corresponds to a cardinal number: Limit Ordinals: $(\omega, \omega_1, \omega_2 \ldots)$, corresponding Cardinals: $(\aleph_0, \aleph_1, \aleph_2 \ldots)$. Although this method also produces new cardinals, it does not produce them directly, they are the result of an ordinal construction. For the two systems to be considered as complete number systems it was necessary to find a direct method for producing infinite cardinal numbers, without reference to methods of ordinal generation.

The method that Cantor introduced to directly generate new infinite cardinal numbers is via the use of the power set function. To recall, if α is a set with β elements then $\wp(\alpha)$ will be a set with 2^β elements, and $2^\beta > \beta$. Here we have a direct method of producing new cardinal numbers. It can be shown that this holds for infinite cardinal numbers, so $\wp(\aleph_0) = 2^{\aleph_0} > \aleph_0$. In general, if $\aleph_\alpha$ is an infinite cardinal number, then $\wp(\aleph_\alpha) = 2^{\aleph\alpha} > \aleph_\alpha$.[19] Having established this separate method, the question as to the relation between these two number systems can be addressed.

The obvious choice would be to make the two systems completely commensurate with each other. This could be achieved if $\wp(\aleph_0) = \aleph_1$, a formulation of Cantor's Continuum Hypothesis, or generally if $\wp(\aleph_\alpha) = \aleph_{\alpha+1}$. But it turns out that the only thing that can be conclusively decided about $\wp(\aleph_0)$ is that it has a cardinality greater than $\aleph_0$. This minimal determination can consistently be strengthened, both the Continuum Hypothesis and its generalization can be asserted, but so can almost any other value of $\wp(\aleph_0)$. Whereas Cantor saw this as a problem within the system of set theory, the failure of set theory to form a closed system conditioned by a single set of rules, Badiou sees it as its saving grace. This realm of undecidability opens up an immanent space within set theory for the appearance of novelty, and for the subject to act on this novelty. It is Cohen's theory of forcing, proving that the Continuum Hypothesis is independent, which opens up this possibility.

If the Continuum Hypothesis holds, then $\wp(\aleph_0)$, the set of all possible subsets of countable, natural, numbers is exhausted by the ordered methods of construction deployed by ordinal generation: $\wp(\aleph_0) = \aleph_1$, or $\wp(\omega) = \omega_1$. The question posed by this hypothesis is: what would it mean to think of infinite subsets of the natural numbers that were not *constructed* according to the ordinal rules of generation? The intuitive response would be that such sets would, in someway, embody a lack of order.

One possible argument would be that the existence of such sets is irrelevant, as they could in no way be effective. The only way that our finite minds can cope with infinite sets is that they *do* embody some order that can be codified in a *finite* way. We can only know such *infinite* sets through their *finite* structure; their members satisfy some property.

18. Potter, *The Philosophy of Set Theory*, pp. 106-107.

19. Potter, *Set Theory and its Philosophy,* p. 262.

This idea recalls the common philosophical theme of duality; a set has its intrinsic ordinal structure, and its purely extrinsic cardinal magnitude: an intensive form and an extensive content. At the finite level these two aspects are indistinguishable and identical, but at the infinite level things change. The Continuum Hypothesis states that the formal aspect takes precedence at the infinite level; we can only discern infinite sets that embody some constructible order. The extensive cardinal magnitude is only accessible through this structured order. If we assert that a non-constructible set can exist, for example if there exists infinite subsets of ω which do not belong to the second ordinal number class (II), how can we have access to them without recourse to some constructive property?

In order to exploit the potential of non-constructible sets a formal approach to sets that lack order must be developed. The Axiom of Choice provides such an approach, by developing a concept of free choice that is independent of any criteria of choice. This axiom affirms freedom and chance, it does not necessarily posit non-constructible sets, but it allows for our manipulation and use of them should they exist.

In this section I have tried to show how Badiou's approach to ontology in *Being and Event* attempts to answer two fundamental questions: how an ontology based on the 'one is not' is possible, and, now, how this ontology is not sterile, it has the potential for real novelty. Novelty can be generated immanently within a situation, due to the minimal relation between a set and its power set, or between a situation and its state representation. All that can be known is that state representation is greater than the original situation, the extent of this excess can never be *known*. But in order to fully exploit this excess of the non-constructible sets, which constitute this undecidable excess of the state, they must be accessible to a subject. The subject must be capable of deploying the consequences of affirming the existence of a certain number of non-constructible sets, without subjecting them to a complete construction or discernment.

In the next section I will introduce the idea of the event, as something that occurs *outside* mathematical ontology. However, the consequences of this event can be expressed as something novel within an ontological situation by a *subject*, this subject depends on the productive *free* affirmation of non-constructible sets. The Axiom of Choice is essential to understanding this free affirmation.

II. THE AXIOM OF CHOICE: INTERVENTION AND THE TIME OF THE SUBJECT

The central role that the Axiom of Choice plays in the subjective realization of an event's consequences depends on Badiou's separation of situations into two fundamental categories, Natural situations, introduced above, and Historical situations (BE 174).[20] Natural situations are *normal*, this normality is provided by their transitive nature. Here

20. These are not the only types of possible situation, Badiou mentions *neutral* situations, 'in which it is neither a question of life (nature) nor action (history)', BE, p. 177. As far as I can tell, he never mentions these situations again.

the relation between a set's extensional, cardinal, existence and its intentional, ordinal, construction share an absolute minimal relation: everything that exists is constructible according to the rules of ordinal generation. Here the Continuum Hypothesis holds, if ω is the presentation of a natural situation, then $\wp(\omega) = \omega_1$ is its state representation. Here every subset, or state representation, is equivalent to a formal production. The state restrictions in a natural situation do not allow anything to 'just happen'. Historical situations, on the other hand, are ab-normal; they represent something *subtracted* from the state representation of a situation (BE 174). They present a *singularity*, something that is presented, but not represented, something that does 'just happen'.

A singular term, for Badiou, is one that is presented in a situation but not represented (BE 99). The subject of an event will always be a finite portion of an infinite procedure that attempts to represent a singular term; this production is the production of a truth. So a singular term is not strictly a presented term that is not represented, it has a *temporal* quality with reference to a subject. It is a term that is *not yet* represented, or one that *will have been* represented.

This is a recurrent theme in *Being and Event*: Badiou makes significant philosophical distinctions by dissecting mathematical proofs and procedures, which are taken mathematically to occur all at once, and imposing a temporal structure on them (BE 410).

This temporalization is important for Badiou's discussion of foundation, which is key to his distinction between Natural and Historical situations. Foundation is a question of origin, in a natural situation the answer is simple and unique: natural situations are founded on the empty set, $\emptyset$. From this set all the others are explicitly generated in a strict order, this order can always be traced back to its foundation. This foundation is, of course, axiomatic. The axiom itself does not justify the empty set's existence it merely asserts it. A situation's foundational element is the one that shares nothing in common with any of its other elements. This indicates its generative function, being the element from which all others are generated. This idea is stated in an axiom, the Axiom of Foundation:

$\forall\alpha\exists\beta\ [(\alpha \neq \emptyset) \rightarrow [(\beta\in\alpha)\ \&\ (\beta\cap\alpha = \emptyset)]]$

To every non-empty multiple there belongs *at least* one element that shares nothing in common with the multiple itself; this is a foundational set. An historical situation is one with at least one non-empty foundational set. Badiou calls such a non-empty foundational set the *site* of an event (BE 175). Clearly such a set shares much in common with the empty set, both are foundational and both are subtracted from the situation, in that they share nothing in common with it. It is these properties that lead Badiou to state that such evental sites are *on the edge of the void* (BE 175). Although they share common properties with the void, or empty set, they are distinguishable from it, if only because they are non-empty. An event is concerned with something other than the proper name of being; it is concerned with the singular specific happening of the event itself.

Badiou readily admits that it is with historical situations that the gap between ontology and thought first opens up (BE 188). Strictly speaking, historical situations can only appear ontologically if these situations are given a temporal dimension. In Cohen's theory of forcing the set that is chosen to extend the standard model of set theory is a set whose elements are non-constructible sets.[21] Here, if the initial situation is thought of as ω, and its state representation as all the sets constructible from it, then if α is a non-constructible subset of ω: $\{\alpha\}\cap\omega = \emptyset$. This *looks* like a foundational set, but we must remember that $\alpha\notin\omega$, and is therefore not foundational. The next move is typical of the kind of temporality that Badiou is introducing. This *potential* site does not belong to the initial situation, but it *could* be added to it. The new initial situation would be $\omega\cup\alpha$, it is clear now that $\alpha\in\omega\cup\alpha$, but equally clear is that $\{\alpha\}\cap\omega\cup\alpha = \alpha$. So *before* its addition to the situation it only satisfied one aspect of foundation, and after its addition it only satisfies the other condition. Only taken as a temporal entity, not solely as a timeless mathematical entity, can the non-constructible set constitute a site.

The decision as to whether this site belongs, or not, is undecidable. To affirm its belonging depends only on the event actually having happened, and the *intervention* of a subject to begin the process of making it belong. The augmented situation does not, therefore, have a site; it is only marked by the trace of a decision. Cohen's theory of forcing produces new situations, which are extensions of the old, but these new situations are natural; they are standard *transitive* models of set theory.[22] To maintain a situation as historical is to keep a process of forcing continually open by focusing on the immanent subject within the situation.

Here the temporal aspect is emphasized again. After a subjective intervention, a decision on the undecidable belonging of a site to a situation, the state of this situation is still that of the old situation *prior* to this intervention. It is the work of the subject to play out the consequences of their intervention through a constant *fidelity* to their conviction that the event occurred. The post-evental state is never fully completed, as the infinite task of the finite subject to extend the state of the situation can never be completed.

The entire theory of the event rests fundamentally on this situated and temporal appropriation of set theory. This is Badiou's philosophical use of ontology, the concepts of the individual inhabitant of a situation, and therefore the subject are *not* mathematical/ontological concepts (BE 411). Cohen's theory of forcing is developed 'in the absence of any temporality, thus of any future anterior, … [to] establish the ontological *schema* of the relation between the indiscernible and the undecidable' (BE 410 my emphasis).

This helps to explain Badiou's peculiar matheme of the event. The matheme of the event is also *not* an ontological statement; it explicitly covets inconsistency. Badiou calls the event the ultra-one and formalizes it in the following way:

$e_x = \{x\in X, e_x\}$

Here, e_x is the event occurring at the site X and it presents not only all its elements, $x\in X$, but also itself. Badiou's use of the Axiom of Foundation makes such a set impossible within consistent mathematical ontology; it is being's prohibition of the event (BE

21. Cohen, *Set Theory and the Continuum Hypothesis*, p. 110.
22. Cohen, *Set Theory and the Continuum Hypothesis*, p. 130.

190). Self-belonging is forbidden within a system of set theory that adopts the Axiom of Foundation. The matheme acts as an inconsistent supplement outside of ontology that lets the subject know that its task is never complete. The task of the subject is to make the truth of the event consist within a situation, to build the relation between the indiscernible and the undecidable (BE 428). In set theoretical terms, the generic extension of a situation, which utilizes non-constructible/indiscernible sets, can decide previously undecidable statements. The key example is the proof of the independence of the Continuum Hypothesis, by demonstrating that there is a consistent situation in which this hypothesis fails. For Badiou, this process is experienced immanently from within the situation, a subject whose endless task is motivated and completed by this external supplement.

Central to the philosophical understanding of an individual or subject's experience within a situation is the Axiom of Choice. It provides not only the potential of an individual to become a subject through an *intervention*, but also the means to maintain subjectivity indefinitely, through the continued *fidelity* to an event.

a) The Axiom of Choice

Intervention is the *illegal* naming of an event, the wager and declaration that something, the event, *has* happened (BE 205). The *choice* of this name is not recognized by the current situation, it is a non-choice for the state (BE 205). The current state restrictions do not encompass the name of the event; this means that the presentation of the name is not constructible according to the current state laws. The name does not conform to any state law of representation. By declaring that an event has occurred, and thus naming it the state apparatus is interrupted and a subject is born.

The potential subjects of a situation are the individual inhabitants who occupy it. This potential for subjectivity is what elevates man, as rational, above the merely animal (E 58-9, 132). It is dependant on their use of mathematics, especially the Axiom of Choice, which makes them capable of intervention. This capacity is hard to define and it seems to involve the coincidence of many classical ideas: rationality, freedom, order and chance. What is interesting is that this capacity can be exercised, to the detriment of the individual, in an autonomous fashion, but it only transforms an individual into a subject when supplemented by an event (BE 230-1). I shall return to this point in the next section.

In the previous section it was the declaration that the site belonged to the current situation, which made it a foundational set, albeit only in a temporal sense. This is the decision of intervention that marks the beginning of the historical transformation of a situation. The subject chooses to affirm the event, and names its site (BE 205). Before the intervention the event occurs, later the subject affirms this event by naming its site: thus only together, an event coupled with a subjective intervention, can a foundation be established. Initially the event is undecidable, it is unpresented in the site, and after its nomination it is illegal at the level of the state representation. It will be the labour of the subject to make this illegal choice legal, to make the truth of the event consist.

The very term illegal states something outside the law, here in an ontological situation that corresponds to rules of construction. An illegal presentation would be the presentation of something not controlled or constructed according to some clear rule. This idea was introduced above with the idea of non-constructible sets. All constructible sets are at base pure extensive multiples, but they all also posses an intrinsic definition, a condition which all its members satisfy. A non-constructible set is one that cannot be given such an intrinsic definition, it can only be considered extensively. In some sense the *laws* governing constructible sets are seen as necessary if any manipulation of infinite sets is to be meaningful. They are the conceptual tongs by which infinite sets can be accessed and manipulated. No such tools are available for non-constructible sets, so either they are not intelligible entities, or they are inaccessible, or there is another way in which they can be accessed. This is what the Axiom of Choice provides, a non-conceptual means of *choosing* and manipulating non-constructible sets. If the laws of constructible sets *govern* and *dictate* the choice of elements in a set, then the Axiom of Choice states that it is possible to choose in an *unrestricted* way: the choice can be unrestricted, free and arbitrary.[23]

The theory of set theoretical forcing works by selecting a set of non-constructible sets to add to a given situation, to expand the number of possible sets constructible within the situation. [24] This initial selection corresponds to the subject's nominative intervention. After this addition the number of possible sets constructible from this new, extended, situation increases. The state representation of the situation is now capable of deciding things which were previously undecidable (BE 416-7). This extension of the state representation, based on the newly chosen and affirmed addition to the situation, does not occur all at once, nor is it ever fully completed. Mathematically it does happen all at once, based simply on it being possible, but within Badiou's philosophy the procedure of extending a situation occurs slowly. The subject is both what produces this slow extension, and the extension itself; the subject is a finite portion of a truth procedure.

This temporal extension of the mathematical procedure is sustained by the subject's fidelity to the event. The impetus to carry on the slow and laborious procedure is given by the meta-ontological matheme of the event: $e_x = \{x \in X, e_x\}$. The matheme has two terms, the elements of its site and its name. These two terms drive subjective fidelity: a fidelity to the subject's *choice* of affirming the site, and a fidelity to the name of the event.

The formal definition of the Axiom of Choice states that if a set exists it is possible to construct a new set by selecting a single *arbitrary* element from each of the subsets of the original set. To give an example, the subsets of a set β constitute the power set of β, $\wp(\beta)$. Now there exists a new set, defined by a choice function, which selects one element from each of the elements of $\wp(\beta)$. At the finite level there is no need for this axiom, take $\alpha = \{a, b\}$, then $\wp(\alpha) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. There are only two possible sets constructible by 'choice', which do not already appear in $\wp(\alpha)$: $\{a, b, b\} = \{a, b\}$, or $\{a, b, a\} = \{a,$

23. Tiles, *The Philosophy of Set Theory*, pp. 190-191.
24. Tiles, *The Philosophy of Set Theory*, pp. 186-187

b}. At the finite level there is no *free* choice, all such sets coincide with one of the initial set's constructible subsets.

We can see that the Axiom of Choice is operating to extend the scope of the Power Set Axiom; it is trying to create, or name, *new* subsets. If only constructible infinite sets are allowed then the limitation on choice extends to the infinite level. A supposed 'choice' function would coincide with a constructible subset; freedom would be subordinate to the law.

The power set function marks the excess between a situation and its state representation. If this excess is legally conditioned by the restrictions of construction then it forecloses the individual inhabitants of a situation against novelty. In order to interrupt this legal conditioning an illegal declaration must be made, one which affirms freedom, accesses the novelty of the non-constructible and deploys the consequences by extending the given situation. But the Axiom of Choice does not arbitrarily affirm the existence of *all* non-constructible subsets; it affirms the existence only of those that it chooses. It allows for a certain *controlled* anarchy, although it affirms and introduces chance it does so in a selective and *ordered* way.

A consequence of this ordered introduction of chaos is that the axiom has a number of significant consequences. For example, the Axiom of Choice is equivalent to stating that every set can be well ordered.[25] This means that every set can be put into a one-to-one relation with an ordinal number, which means that it can be constructed. This might seem to contradict the fact that the axiom seems to introduce non-constructible sets, but what has to be noted is that constructability and non-constructability are *relative* to a situation. This is due, partly, to the fact that the ordinal numbers do not in their totality form a set: there is no set of *all* ordinal numbers.[26] This, for Badiou, means that although there are natural situations, there is no such thing as Nature in its totality; Nature does not exist (BE 140-1). There is no ultimate level that could either absolutely affirm or deny the non-constructible. Where non-constructible sets are affirmed to exist they represent a symptom of the situation's limits. The question is whether this is a desirable symptom; is it a symptom of disease? Should the non-constructible be viewed as deficient and lacking, or should it be affirmed and incorporated?

The limit ordinals code, in their structure, a certain degree of complexity by defining all the possible sets constructible from a certain number of rules. Every situation is conditioned by a limit ordinal, which restricts the degree of constructed complexity.[27] If only constructible sets can appear within a situation there is no problem, but the Axiom of Choice can force sets to appear in a situation that present a greater degree of complexity than the current situation can condition. Therefore, in this situation the construction of these sets cannot be known and they appear random and non-constructible. A further ordinal external to the situation could provide a rule for construction, but it is not immanently available to an inhabitant of the current situation.

25. Potter, *Set Theory and its Philosophy*, p. 224.
26. Potter, *Set Theory and its Philosophy*, p. 181.
27. Tiles, *The Philosophy of Set Theory*, p. 187.

The Axiom of Choice also greatly simplifies cardinal arithmetic, and also dictates that every infinite cardinal number is an aleph.[28] If we recall, the rules of ordinal generation produce a limitless succession of ordinal numbers, each limit ordinal being the first number to be associated with a new cardinal number, and these cardinal numbers are called alephs. What the above idea suggests is that there *is* a minimal relation between ordinal and cardinal number production; it might not be the strict relation of the General Continuum Hypothesis: $\wp(\aleph_\alpha) = \aleph_{\alpha+1}$. But there is, nevertheless a relation, the freedom of the Axiom of Choice still chooses within limits. Every cardinal is always equivalent to some ordinal.

In this section I have explored three different uses of the Axiom of Choice. First, choice is subordinate to the current law of the situation. Anything that appears to be a free choice in fact coincides with a constructible and legal part of the current situation: nothing new is produced. Second, a subjective intervention claims that certain freely chosen non-constructible sets belong to the situation. They *extend* the current situation through the novel constructions they allow. Third, freely chosen non-constructible sets are accepted as non-constructible and novel within the current situation, but a *new* situation is posited in which they are constructible. Only the second scenario, the subjective scenario, allows the illegal sets to retain their non-constructible status. Although, during the course of a truth procedure, the *names* of the non-constructible sets *become* legal, their non-constructible nature remains. The constructible and non-constructible co-exist. In the first case non-constructability is denied, and in the third case it is a *problem* solved through the introduction of a *new* situation with *new* rules of construction.

The random aleatory character of non-constructible sets are not considered a deficiency by the subject, their chance nature is affirmed. This idea that the subject *extends* a situation rather than creating a *new* situation is important to Badiou (BE 417). A new situation suggests that the subject performs a transcendent role. In such a transformation the subject gains access to an ordinal number outside and beyond the current situation in order to solve the *problem* of a multiple's non-constructability. This new ordinal is of sufficient complexity to define the construction of the previously non-constructible multiple. With Badiou's theory the subject remains firmly within the current situation and transforms it immanently. His only appeal to a meta-mathematical concept is to the matheme of the event. The matheme does not provide a transcendent multiple necessary for the transformation, but opens a temporal space in which the subject operates.

Although the full theory of set theoretical forcing is necessary to appreciate Badiou's subject, I believe that it is with this concept of freedom, motivated by the Axiom of Choice, that Badiou makes his most significant ethical distinctions. The three distinctions, made above, all reappear in Badiou's book on ethics. The misuses of freedom in being subordinate to the law, or attempting to transcend a given situation correspond to Badiou's categories of Terror, Betrayal and Disaster. The good is entirely defined by a correct subjective operation. But what if a correct subjective operation undermines the

28. Potter, *Set Theory and its Philosophy*, p. 266.

freedom of the subject/individual itself, what kind of subject would that be?

b) Ethical Categories

Badiou's theory of ethics focuses entirely on a clear distinction between Good and Evil, with Evil only being possible on the basis of the Good (E 16). The Good is defined as what results from a correct subjective response to an event. This involves the occurrence of an event, and the production of novelty/truth within the situation, as the result of an initial subjective intervention and their subsequent faithful labour. Evil occurs only when some aspect of this complex arrangement goes wrong (E 60). Here, the presupposition that I find difficult to accept is that all events, and subjective responses are fundamentally Good. This might not seem problematic, affirming the creative free expression of a subject, who extends the possibilities of a situation through the production of truth, but these common themes of subjectivity, freedom and truth are completely transformed in Badiou's system. They no longer have their everyday intuitive appeal. Rather, the distinction between Good and Evil is too convenient, and seems *derived* from the system of *Being and Event* rather than expressing something true. The theory of ethics developed by Badiou seems to be *consistent* with his systematic philosophy rather than with experience.

For me, Badiou's ethics appear to be based too strongly on the notion that the theory of forcing, borrowed from Cohen, is essentially a liberating operation. In providing the final proof of an axiom's independence from the standard axiom system, set theory is liberated, or emancipated, from the constraint imposed by it. Badiou presupposes two things: emancipation from a given axiom liberates the formal system from a constraint, the system becomes more open as a result, and the potential of a future subject remains intact after a process of forcing (BE 416). It is this second idea that I want to particularly concentrate on. As I have demonstrated during the course of this essay, the Axiom of Choice is essential if a subjective response is to be possible within a situation. One of the aims of developing the theory of forcing was to prove the independence of the Axiom of Choice, that is, to force a situation in which it fails.[29] Badiou calls the future anterior situation when a truth *will have been* forced, the post-evental situation. This is an almost Kantian 'as if' projection, to consider a situation *as if* the truth had been completely forced.[30] What is the post-evental situation if the Axiom of Choice has been forced to fail by a subject adhering to the strictures of set theoretical forcing in strict fidelity to an event? This situation will be one in which it is impossible for a new subject to arise, the individual will be stripped of their freedom. The Axiom of Choice won't be in a dormant state subordinate to the law, as it is in the restrictive constructivist's situation. The Axiom of Choice, and therefore the individual's freedom will have been an inconsistent principle.

In order to explore this idea more fully, I will examine the ethical categories of Ter-

29. Cohen, *Set Theory and the Continuum Hypothesis*, pp. 136-142.

30. Badiou, 'Truth: Forcing and the Unnameable' in *Theoretical Writings*, p. 127.

ror, Betrayal and Disaster in order to show that none of these covers the possibility I have suggested. The forcing of the failure of the Axiom of Choice is a positive example of an undesirable event and a subsequent, fully legitimate, undesirable subject. This, I think, posses a significant problem for the simple division of Good and Evil in Badiou's philosophy.

Badiou finds the *simulacrum* of an event the most dangerous form of evil due to its formal similarity to a true event (E 72). The simulacrum deploys its pseudo-subjectivity in the form of terror. The simulacrum is potentially the most interesting form of evil as it allows for degrees of terror. The concept rests firmly on the Axiom of Choice and intervention, here though, the intervention is the intervention of an individual. What the individual names as the site of the event, is only what superficially appears to be the site. Thus the individual remains an individual, and does not become a subject.

The importance of the site, prior to the subjective intervention, is that it should share nothing in common with the current situation. If S is the situation and X the potential site of an event: $S \cap X = \varnothing$, X is on the edge of the void. The site is important, as sharing nothing with the situation it is equally addressed to the whole situation, there is no privileged subset of the situation that could claim special access to the event (E 73). In the case of the simulacra this supposed site is not empty, it is not on the edge of the void (E 73). Here the intervention is not based on a radical emptiness of the site, but on plenitude.

Essentially if the intersection, $S \cap X \neq \varnothing$, is not empty then this intersection constitutes an already existent subset of the situation. A constructible subset already exists that represents, at least partially, the supposed site of the event. The event can then become *identified* with an already established group, in his example of Nazi Germany Badiou gives the example of the concept of German racial purity (E 73). The question that arises is, that although the intersection is not empty, would it be empty if the identified subset where removed? For example, if $S \cap X = \alpha$, would $S \cap (X\text{-}\alpha) = \varnothing$ and, further more, is $(X\text{-}\alpha)$ non-empty? Here is the danger inherent in simulacra, as if both of these conditions are fulfilled, then $(X\text{-}\alpha)$ could be a genuine site of an event. Here there are two possible types of terror, a terror that hijacks an actual event and one that does not.

Formally or mathematically speaking the simulacra does not occur. If $(X\text{-}\alpha)$ were a genuine site, then so would X. The appearance of α would be dismissed from the formal mathematical approach, it would be seen as the mere repetition of a constructible set and removed or ignored. But within the temporal philosophical approach, developed by Badiou, this repeated subset causes immense problems.

The pseudo-subject of a simulacrum might well be generating true novelty, but the organization of this novelty under the name of a privileged subset of the original situation strips it of its truth. The address is no longer universal; it is addressed to the preordained chosen ones. Their domination of the potentially revolutionary novelty results in a reign of terror. All true subjects are open to the potential for their event to become a simulacrum, to identify its message with a predetermined group or class.

Betrayal is possibly the simplest category of Evil; it is a renunciation of one's participation in a truth procedure, and therefore a renunciation of one's subjectivity. This renunciation cannot be in the form renouncing one's *interest* in a certain cause, but must reject the very cause itself as having ever been significant (E 73). The Axiom of Choice, again, plays a central role. Here, with respect to the truth that I used to believe in, I claim that its novelty and uniqueness were merely derivative. I affirm in my renunciation that the site, which I took to be composed of non-constructible multiples, was in fact wholly constructible. The individual accepts that their freedom is only ever apparently free from their own perspective; in actuality it is subordinate to the law. Their freedom, embodied in the Axiom of Choice, is actually nothing more than a theorem entailed by a universe restricted to constructible multiples: the Axiom of Choice loses its vital axiomatic status (E 305-7).

Finally, the Disaster is what Badiou calls an attempt to name the unnameable. Here the full power of the Axiom of Choice is deployed, in an attempt to eradicate the singularity of the event in favour of the pure autonomy of the individual's freedom. There are two ways for the Axiom of Choice to deal with the possible appearance of non-constructible sets. The first, forcing, is the method chosen by the subject, where the non-constructible aspect of an event's site are made to consist in a situation. The second uses the fact that the Axiom of Choice allows all sets to be well ordered. The ordinal required to well order the non-constructible sets are not available within the limitations of the current situation. This ordinal is an unnameable for the situation, and a disaster for truth is when the individual appeals to his freedom, in the form of the Axiom of Choice, in order to name this unnameable. As Badiou claims: 'Rigid and dogmatic (or 'blinded'), the subject-language would claim the power, based on its own axioms, to name the whole of the real, and thus to change the world' (E 83).

The random character of the event, which the subject requires in order to affect an intervention, is abandoned. The individual's free choice is exercised in an isolated and autonomous fashion, which characterizes the event as a problem to be solved. In the new situation nothing of the event is left, or preserved. This is a disaster for truth, rather than affirming the truth of a situation the individuals seek confirmation of their own autonomy and power in an appeal to a transcendent realm. In the mind of God there is no confusion, there is nothing that cannot be constructed, the individual need only make an appeal to this totalized transcendent realm in order to find a solution to the problem of the event.

All of these forms of Evil rely, in one way or another, on the 'misuse' of an individual's capacity for free choice. The individual's inability to correctly deploy the Axiom of Choice, in the face of an event prevents them from making a subjective intervention. But the proof of the independence of the Axiom of Choice clearly falls into the 'correct' use of the Axiom of Choice; it inaugurates a subject through an intervention. It is somewhat bizarre, though not inconsistent, that the Axiom of Choice is a necessary axiom in the forcing of its own failure, but this does not stop it from being a valid instance of set theoretical forcing.[31]

The forcing of the failure of the Axiom of Choice works by adding non-constructible sets of a certain type to a situation. In order for the Axiom of Choice to function in the extended situation, supplemented by these non-constructible sets, it is necessary that all the sets constructible within this situation can be well ordered. For this to be possible the added sets need to be distinguishable from each other given only a finite amount of information. It is possible to *choose* non-constructible sets where this does not happen, well ordering of the constructible sets fails and so too does the axiom of choice.[32] The subject is no longer able to cope with the truth that his intervention affirmed. The subject is not capable, even potentially, of fully deploying the truth of the event.

Badiou's argument that his theory of the subject, modelled by set theoretical forcing, brings a new rationalism to the study of the subject is undermined at this point. This rationalism is based on the subject's ability to cope with events and deploy the consequences. The faith, or fidelity, of the subject is based on the Axiom of Choice as it allows, in the model of forcing the independence of the Continuum Hypothesis, the differentiation of the non-constructible sets from any given constructible or non-constructible set on the basis of a finite amount of information. The *finite* subject's faith is *justified* on the grounds that it can differentiate sets on a *finite* amount of information, regardless of whether it achieves a specific differentiation within its own lifetime. This faith is undermined if such a differentiation is not finitely possible.

The subject that forces an event that undermines their subjectivity and the *rational* power of the Axiom of Choice to manage and produce order has an echo of the sublime about it. In encountering an event of a specific kind the subject experiences something beyond the power of his free rational power to manage. Although here this does not strengthen the subject, but threatens to destroy it. If the subject holds its fidelity to this event it then enters willingly into this nihilistic endeavour. Once the Axiom of Choice has been undermined the minimal relation between the intensive and extensive character of multiples is lost, every infinite cardinal is no longer an aleph. Extensive multiples are no longer tied to intensive multiples, not even to a range of possible intensive multiples. The relative simplicity of the set theoretical universe is somewhat complicated.

31. Tiles, *The Philosophy of Set Theory*, p. 190. Many of the features of the Axiom of Choice's use, especially in the context of Badiou's philosophy, offer parallels with Sartre's concept of bad faith. Here, for example, the Axiom of Choice, as an individual's free capacity to choose, is employed *against* that very capacity, seeking to undermine it. But this use still requires an event to supplement it, unlike Sartre. Closer would be the concept of betrayal, seen above, here freedom denies itself as free reducing itself to a theorem who's results are governed by law. This possible relation between these two thinkers is further complicated by Sartre's later work in the *Critique of Dialectical Reason*, where a similar philosophy of the event is developed. Any substantial investigation of this relation between the Axiom of Choice and bad faith would have to address the question of what happens to the concept of bad faith in Sartre's later writings.

32. Cohen, *Set Theory and the Continuum Hypothesis*, p. 136. Badiou's technical response may be that the set used to force the independence of CH only contained *denumerable* non-constructible elements, whereas the set used to force the independence of AC uses *non-denumerable* elements. This would force Badiou into accepting a limited form of AC, based on *countable* choice, but in *Being and Event* he affirms the full power to the axiom.

I am not sure what the possible consequences of such a subject are for Badiou's philosophy. It does complicate his ethics. A self destructive subject intent on affirming something beyond reason's control could be seen as an unwelcome return of the irrational, no longer considered as inconsistent but as exceeding the power of choice, or as a reintroduction of the sublime and the Other, something which Badiou specifically wants to avoid (E ch2). But this subject is not the product of a misuse of the Axiom of Choice, but one formed according to the model outlined in *Being and Event*. Therefore, to preserve Badiou's ethics this subject must be either denied, it is not a subject, and might possibly constitute a new category of Evil, or it is a subject and its activity is to be affirmed as Good. Both options do not seem to comfortably fit into the framework as it stands. If this subject is affirmed the consequences of the post-evental situation need to be addressed. Although *Being and Event* allows for the indeterminacy of non-constructible sets, their inclusion is limited to those that human individuals can cope with. The individual can only allow forms of presentation that the Axiom of Choice can manipulate, that is, those sets that can be subjected to the individual's rational power. Without this rational capacity Badiou feels that man is reduced to his animal status, and incapable of ethical practice. But in the type of situation and subjectivity described above, it could be argued that the subject is in the process of exceeding his rational limitations, acting in a selfless way in the face of something that he cannot master. Perhaps this is a more fitting figure for the ethical subject, and the post-evental situation, although it never arrives, a more interesting ethical situation?

In conclusion, Badiou's use of set theory, in his conceptualization of the subject, allows him to take a truly original approach to both ontology and philosophy. The mathematical approach gives him the ability to add great clarity and distinction to otherwise similar concepts, such as the name of the void, in general, in the form of the empty set, and those entities on the edge of the void that constitute evental sites. Here Badiou's philosophy is at its strongest, rejecting the problems of systematic philosophy and ontology as an endless problem of grounding by adopting the axiomatic method, and thus explicitly nullifying the problem. The problem of the ground, or the Axiom of the Empty Set, does not recur in ontology, what occurs, instead, are events.

But set theory is also something of a Pandora's box. There are so many clearly defined bizarre entities within this universe that many of the aspects of philosophy that Badiou wants to reject, especially in recent continental philosophy, can return from the realm of inconsistency, where he banishes them, and associate themselves with some of these more unusual and offbeat products of mathematics. In this essay I have introduced the possibility that the independence of the Axiom of Choice could reintroduce themes of the Other and the sublime right into the heart of Badiou's philosophy.

What this proves is not that Badiou's philosophy is a failure but that this approach has a huge potential for productive work, even if this may divert from, or undermine, Badiou's own singular vision for his work. The central place of the subjective in the production of novelty and truth in Badiou's philosophy of events is a position that I think needs to be questioned.

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