BADIOU AND THE CONSEQUENCES OF FORMALISM

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ABSTRACT: I consider the relationship of Badiou's schematism of the event to critical thought following the linguistic turn as well as to the mathematical formalisms of set theory. In *Being and Event*, Badiou uses formal argumentation to support his sweeping rejection of the linguistic turn as well as much of contemporary critical thought. This rejection stems from his interpretation of set theory as barring thought from the 'One-All' of totality; but I argue that, by interpreting it differently, we can understand this implication in a way that is in fact consistent with the critical and linguistic methods Badiou wishes to reject.

KEYWORDS**:** Ontology; Badiou; Formalism

I

Since at least the *Theory of the Subject* of 1982 (comprising seminars held from 1975 to 1979), Alain Badiou has attempted in an unparalleled way to conceive of the political and ontological implications of formalism, subjecting the very constitutive structures of ontological being to the dictates and rigors of abstract mathematics. One of the most significant outcomes of Badiou's thought is his application of formal methods to what has also become an obsession of contemporary continental philosophy, the problem of theorizing the "Event," or the transformative eruption of the essentially unforeseeable *new* into a given, determined situation.[1] According to a problematic

[1] The significance of this problematic of the event goes back at least to Heidegger's discussion of *Ereignis*, the mysterious "event of enowning" that transforms in a fundamental way the basis for whatever is in being; in subsequent discussions, Derrida and Deleuze have each (in different ways) accorded their

already developed and pursued by Heidegger, such genuine novelty demands, as well, a fundamental break with all that can be said with the language of the metaphysical tradition, including all that is expressed or expressible by the "ontological" language that comprises everything that can be said *of* what *is*. For Badiou, in order to develop such a theorization of novelty as such, it is thus necessary first to model the "ontological" structure of being, insofar at least as it can be described, in order thereby to develop a rigorous schematism of what occurs or takes place beyond it.

This attempt to articulate symbolically the advent of novelty which occurs, for Badiou, beyond the limits of "what can be said of being qua being" threatens to put Badiou, like others who have attempted to trace the "closure" of a "metaphysical" language that avowedly determines everything that can be said of what is, in a paradoxical and even self-undermining position. This is the dilemma (familiar to readers of the early Wittgenstein) of the philosopher who would speak of what is by his own lights unspeakable, who would attempt by means of symbolic language to trace the very boundaries of the sayable as such in order to indicate what lies beyond. One sort of solution to this dilemma (which is, of course, not without its own problems) lies in the Wittgensteinian attempt to discern, beyond the ordinary significative function of language in saying, the distinct function of an ineffable "showing" that operates, most of all, where language exceeds its own bounds and thus falls into nonsense. Badiou, however, solves the problem in a very different way, one that suggests a radically different understanding of the significance of formalization itself. For faced with the dilemma of the demonstration of the unsayable, which cannot, on pain of contradiction, amount to a significative use of language, Badiou foundationally and completely disjoins the formalisms of mathematics from language itself, attempting a formalization both of all that is sayable of being *and* of what lies beyond this regime by means of the abstract (and, for Badiou, wholly non-linguistic) schematisms of mathematical set theory. For according to Badiou, where language cannot speak, the formalisms of mathematics, definable purely by their abstract transmissibility, beyond the constraints of any particular language, can nevertheless display the structure of the sayable, as well as the structure of the Event which necessarily lies beyond it.[2]

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different formulations of the "event" a central place in their own critical projects. For Heidegger's conception, which I do not discuss in detail here, see, e.g., M. Heidegger, *Contributions to Philosophy: From Enowning*, transl. by Parvis Emad and Kenneth Maly, Indiana University Press, [1938] 2000; and M. Heidegger, *Identity and Difference*, trans. J. Stambaugh, Chicago, University of Chicago Press, [1957] 2002.

[2] There are potential problems here, insofar as this rigid disjunction between mathematical formalism and language, which indeed solves the dilemma on one level, nevertheless makes it virtually impossible

More specifically, Badiou identifies the axiom system of Zermelo-Fraenkel set theory as defining the regime of ontology, or the possible presentation of what is as such. [3] This interpretation then serves as the basis for his suggestive as well as problematic formal schematism of the Event, which, in *breaking* with these standard axioms at a certain precise point, also locates, according to Badiou, the point at which the ontological order of being is itself interrupted and surprised by the transformative eruption of an essentially unforeseeable novelty. In the more recent *Logics of Worlds*, Badiou continues this analysis with a formal consideration, based this time on category theory, of the primarily linguistic establishment and transformation of the boundaries and structure of particular situations of appearance, or worlds.[4] Here again, the possibility of any fundamental transformation in the structure of a particular, constituted situation depends on a formally characterized effect of ontology, a kind of "retroaction" by means of which an ontologically errant set-theoretical structure allows what was formerly utterly invisible suddenly to appear and wreak dramatic substantive as well as structural changes.

In both of Badiou's major works, the interpretation of structures that have been considered "foundational" for mathematics thus operates as a kind of formalization of the limits of formalism themselves, which in turn yields radical and highly innovative interpretations of what is involved in thinking both the structuring of situations as such and the possibilities of their change or transformation. One of the most far-ranging of these innovative consequences of the interpretation of formalism, as Badiou points out, is that it renders the *infinite* mathematically (and hence, according to Badiou, *ontologically*) thinkable. In particular, Cantor's theory of multiple infinite sets, which is at the very foundation of contemporary set theory in all of its versions, yields a well-defined mathematical calculus which allows the "size" or cardinality of various infinite sets to be considered and compared. This symbolism has, as Badiou emphasizes, profound consequences for the ancient philosophical problem of the one and the many, and hence for any systematic consideration (mathematical, ontological, or political) of what is involved in the formation and grouping of elements into a larger whole. [5]

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for Badiou to justify his own reflexive (and, necessarily, it seems, *linguistic*) interpretations of the schematisms themselves.

[3] Alain Badiou, *Being and Event*, trans. Oliver Feltham, London, Continuum, [1988] 2005. (Henceforth: *B&E*).

[4] Alain Badiou, *Briefings on Existence: A Short Treatise on Transitory Ontology,* trans. Norman Madarasz, SUNY Press, [1998] 2006.

[5] It is thus possible to see in the radical consequences of Cantor's thinking of infinite totalities the specific limitation of Levinas' thought about the relationship figured in the title of his book *Totality and Infinity* (E. Levinas, *Totality and Infinity: An Essay on Exteriority*, trans. Alphonso Lingis, Pittsburgh, PA, Duquesne

By far the most mathematically and conceptually radical consequence of this definition of the set as a "many which can be thought of as one" was Cantor's theorization of the *infinite series* of natural numbers (1, 2, 3, …) as comprising a single "completed" set. With this single, bold, theoretical step, Cantor reversed thousands of years of theory about the infinite, stemming originally from Aristotle, which had held that such infinities as the series of natural numbers could only be "potential" infinities, never existing as actually completed wholes.[6] Moreover, with the same gesture, Cantor also suggested the existence of a vast open hierarchy of 'completed' infinite sets, each bigger than the last, beyond the set of natural numbers itself. For, as he quickly showed, the definition of a set already allows us to consider its *subsets*, those sets that are comprised only of some of the original set's elements. We can then consider the *power set*, or the set of all subsets; and as Cantor showed with the theorem that still bears his name, the power set will always be strictly larger – will contain 'more' elements – than the original one. By repeatedly applying the power set operation to the original, infinite set, we thus obtain an apparently boundless hierarchy of larger infinite sets, whose relations of size or cardinality can then be discussed and compared.[7]

At one stroke, Cantor thus both radically transforms mathematical thinking about the status of infinity and creates contemporary set theory by allowing that arbitrary multiplicities can indeed be considered to be well-defined and actually completed

University Press, [1961] 1969) which in fact figures this relationship not as a conjunction but as an exclusive disjunction. This is why, for Levinas, the phenomenological "openness" to infinity, for instance in Descartes' argument, always points the way to a "infinite transcendence" that lies outside the possible survey of any totality. If Cantor has succeeded in his formalization, however, this opposition is by no means demanded by the thought of the infinite, which can indeed yield a doctrine of infinite totalities; and hence, as Badiou argues, Cantor's innovation can be the specific agent of the historical passage of thought about infinity from the categories of the mystical, transcendent, or religious (in which it still falls for Levinas) to a thoroughly de-mythologized and "atheistic" treatment of the role of the infinite in a finite human life.

[6] For more on this history, see A. W. Moore, *The Infinite*, Routledge, 2001.

[7] These innovations already led Cantor to pose what is, today, still one of the most notorious unsolved problems in all of mathematics. This is the problem of the status of the "continuum hypothesis," or of the relationship between the size of the 'first' infinite set (the set of natural numbers) and that of the set of *real* numbers, or of discrete points on a continuous line. As can easily be demonstrated by means of diagonalization, the continuum is indeed larger than the set of naturals; the problem (to the solution of which Cantor labored for decades, but in vain) is how much bigger it is. The *continuum hypothesis* holds that the continuum has the size, or cardinality, of the very "next" infinite set beyond the set of natural numbers (which has the 'first' transfinite cardinality, aleph-naught). As we now know, owing to decisive independence results proven by Gödel in 1939 and Cohen in 1963, the hypothesis cannot be proven (Cohen) or disproven (Gödel) from the standard ZFC axioms of set theory.

wholes. Yet how big an infinite many indeed "can" exist as a one? Is there any limitation to the size of successive infinities formed by means of the power set operation, or does the hierarchy itself extend without any boundary? And what, then, should we say about the existence and size of this *whole* infinite hierarchy of infinite sets? As Hallett (1986) has recently shown, Cantor's own thought about these questions is motivated, at least in part, by theological considerations, which led him to believe both that the well-defined infinite sets of the naturals, or of the reals, can exist as wholes in that God can indeed group them all together as unified sets (even if finite agents cannot) and that the whole infinite hierarchy of infinite sets forms an "unincreasable" totality that cannot be treated mathematically at all, what Cantor called the *Absolute*. This Absolute infinity is, for Cantor, "unreachable by any determination;"[8] it thus inherits the position occupied in earlier theories, for instance those of Aquinas and the scholastics, by an absolute divinity whose magnitude is incapable of numerical or any other positive specification. Thus, despite the radical innovation of Cantor's theory in positing the actual existence as a set of *any* multiplicity (be it finite or infinite) that can indeed "be thought as one," his understanding of the Absolute leads him effectively to posit that there are indeed multiplicities –- most notably, the multiplicity of *all* sets, or what we might otherwise call the "set-theoretical universe" as a whole -- that are "too big" to be thought of as sets at all. In a later text, Cantor termed such "too big" multiplicities "inconsistent multiplicities" – reflecting the intuition that they indeed *cannot* (consistently) be thought together as Wholes – reserving the term "set" for the smaller "consistent multiplicities" that can indeed be thought as one.[9]

Although Cantor's motivations in holding the Absolute – or the set of all sets – to be indescribable mathematically, on pain of contradiction, was primarily theological, subsequent developments in set theory themselves would bear out his intuition in a striking and deeply suggestive way. The subsequent development of a series of far-reaching set theoretical paradoxes appeared to show that it is indeed *impossible to conceive* of a "set of all sets," or of certain other related multiplicities, as completed wholes, without encountering contradictions. The first of these paradoxes was the one already discovered in 1897 by Cesare Burali-Forti, which appeared to show that the set of all orderable or "ordinal" numbers, considered as itself an ordinal number, must be both larger and smaller than itself. Just four years later, Russell's paradox

[8] Hallett (1986), p. 39 (quoting from p. 176 of Cantor (1883))

[9] "When … the totality of elements of a multiplicity can be thought without contradiction as 'being together', so that their collection into 'one thing' is possible, I call it a *consistent multiplicity* or a *set*.' (1899 letter to Dedekind, quoted in Hallett (1986), p. 34).

would put a closely related result in vivid form, as the demonstration of the contradiction that follows necessarily from the supposition that there exists a set of all sets that are not members of themselves.

In the first pages of *Being and Event*, Badiou describes set grouping or unification as the result of a fundamental operation of "counting as one" which forms an indifferent multiplicity into a structured one that can indeed be "counted" or presented as such.[10] The outcome of this operation is the formation of anything that can indeed be understood as a presented whole with any structure whatsoever; all investigation of the effects of structuration and formation on any existing situation can therefore proceed from an investigation of the possibilities and properties of this fundamental "count-as-one." Following Cantor's own terminology, Badiou calls the successful result of this operation – an actually existing set, be it finite or infinite – a "consistent multiplicity;" before the count-as-one, there are only "inconsistent multiplicities" which precede any formation into ones, and so indeed cannot be thought or conceived mathematically (or ontologically) at all.[11] The distinction between consistent and inconsistent multiplicities, so described, is to be regulated, Badiou holds, by an axiom system that implicitly defines which sets can exist (and hence which multiplicities cannot be grouped as sets at all).[12]

This appeal to the axiomatic structure of set theory and the consequent need to avoid the formation as sets of any of the "too-large" inconsistent multiplicities forms the backdrop to the first and most general of the axiomatic "decisions" that comprise Badiou's own systematic ontology. This is the decision of the "non-being of the one" from which, as Badiou says, his "entire discourse" originates.[13] According to this decision, "the one *is not*;" fundamentally, there are only multiples and multiplicities. These multiples can indeed, in general, be grouped into ones by the action of structure, or the "count as one"; what cannot exist, however, is the "One-All" or universe that would result from the grouping together of everything that exists.

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[10] *B &E*, p. 24.

[11] There are actually two distinct questions here: i) of what (if anything) "precedes" the operation of the count-as-one; and ii) of what (if anything) cannot be counted as one at all, on pain of contradiction. Cantor uses 'inconsistent multiplicity' primarily to describe ii); since he lacks any clearly formulated conception of the set grouping 'operation' itself, he does not explicitly extend this usage to i). However, Badiou argues (pp. 41-43) that given an axiomatic definition of the grouping operation, we can indeed identify the two senses of "inconsistent multiplicity." For more on these topics, see Paul M. Livingston, *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism*, Routledge, 2011, especially chapters 8 and 9.

[12] *B &E*, pp. 29-30.

[13] *B &E*, p. 31

Badiou presents this axiomatic decision against the One-All as a fundamental rejection of the legacy of Parmenides and, indeed, of the entire ontological tradition he founded.[14] But although his rejection of the One-All is, like other significant decisions, axiomatic, Badiou does not hesitate to give a justification for it in terms of set theory. This justification turns on Badiou's interpretation of Russell's paradox and the related paradoxes, which led Russell and subsequent logicians to seek devices to prevent the possibility of forming the problematic sets.

The Russellian 'theory of types,' is one such device, as are the axioms of foundation and separation enshrined in the now-standard axiom system of Zermelo-Fraenkel set theory. The intent behind all these devices is to prohibit the *self-membership* of sets; in other words, they all prevent, at a basic level, the possibility of a set belonging to itself. In this way, the "paradoxical multiplicities" or sets leading to contradictions are immediately prohibited; so, also, is the 'total set' or set of all sets.[15] Badiou follows the tradition of logicians in both prohibitions, holding that since the existence of a contradiction would "[annihilate] the logical consistency of the language,"[16] the problematic sets cannot be formed, or in other words that the problematic multiplicities, including the multiplicity of all multiplicities, do not exist as Ones. The universe described by language is thus essentially and fundamentally *incomplete*; this result provides formal grounds for the basic decision "against the One-All," which, Badiou holds, must be maintained by any systematic, axiomatic theory of being itself. Thus:

> Inconsistent or 'excessive' multiplicities are nothing more than what set theory ontology designates, prior to its deductive structure, as pure non-being.
>
> That it be in the place of this non-being that Cantor pinpoints the absolute, or God, allows us isolate the decision in which 'ontologies' of Presence, non-mathematical 'ontologies', ground themselves: the decision to declare that beyond the multiple, even in the metaphor of its inconsistent grandeur, the one is.
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> What set theory enacts, on the contrary, under the effect of the paradoxes – in which it registers its particular non-being as obstacle (which, by that token, is the non-being) – is that the one is not. (p. 42)

[14] It is "a decision to break with the arcana of the one and the multiple in which philosophy is born and buried…", (*B&E*, p. 23).

[15] For, such a set, if it existed, would be (since it would be a set) a member of itself.

[16] *B&E*, p. 41.

Badiou is indeed right to hold that the paradoxes establish a fundamental result, transformative for all systematic consideration of the one and the many, in establishing the fundamentally problematic status of the attempt of traditional metaphysics to think an unproblematically unified totality, the traditional "One-All" of the universe of all that exists. However, with respect to the formalisms themselves, there is an important alternative here which Badiou does not so much as acknowledge. For as some logicians have more recently emphasized, it is not at all the case that the Russell paradox, for instance, simply *forces* the decision against a One-All or a set of all sets. For we may, by means of various alternative devices, *affirm* the existence of the total set while nevertheless acknowledging the Russell paradox. One way to do this is to permit axioms allowing the existence of self-membered sets, including the total or 'universal' set, while still prohibiting the problematic Russell set itself.[17] Alternatively, we may tolerate the existence of the Russell set and the other contradictory sets by *allowing the existence of certain contradictions* – contradictions that characteristically arise in the course of thinking, or talking, about the limits of a totality in which the act of thinking or talking *itself* is a member.[18]

In fact, the choice to affirm the existence of the totality, and thus to uphold the completeness of language in its capability of speaking the All, defines an alternative critical orientation, one which is also heir to the paradoxes but strikingly at odds to Badiou's own. We can see this difference particularly clearly, indeed, in relation to the status of another result that figures directly the consequences of self-belonging and diagonalization, Gödel's (first) incompleteness theorem. As we have already seen, although the theorem is usually called the "incompleteness" theorem, it in fact faces us with a decision *between* completeness and consistency. Affirming the consistency of the formal system in which it is formulated (for instance, *Principia Mathematica*), we may take it that the result shows that this system is *incomplete*: that is, that "there are truths" that it cannot prove (such as, for instance, the truth of the statement of the Gödel sentence itself). However, we may also just as well take it to show that the system is *inconsistent*, i.e. that there is some proposition, A, of which it proves *both* A and its negation. In this way we may preserve the completeness of the system (of *PM*, or by analogy, the system of language itself in its capability to *say* everything) at the cost of determining it to contain inconsistencies. Of course, this is not the route usually taken, since it has usually been assumed that a contradiction ruins the integrity of any system, since "from a contradiction anything can be proven." However, as we shall

[17] See, e.g., T.E. Forster, *Set Theory With a Universal Set*, Oxford, Oxford U. Press, 1992.

[18] See, e.g., Graham Priest, *In Contradiction: A Study of the Transconsistent*, 2d Edition, Oxford U Press, [1987] 2006, especially chapter 2, and Graham Priest, *Beyond the Limits of Thought*, Second Edition, Oxford, Clarendon Press, 2002.

see, this is by no means necessarily so, and depends in detail upon the structure of the logic of proof that is employed. In any case, and even more significantly, although we *may* make the decision for consistency and incompleteness, *or for* completeness and inconsistency, neither one is mandated by the formal system itself. For – and this is the precise content of Gödel's *second* incompleteness theorem – it is impossible for a formal system to prove its own consistency; it is thus always possible to take it, and impossible to foreclose the possibility from within, that it may contain inconsistencies.

More generally, then, we might put the situation as follows. It is not in fact the case that the implications of the Russell paradox or any of the related semantic paradoxes immediately force us to reject, as Badiou claims, the "One-All." The effect of the paradox is rather to *split* the One-All into two interpretive hypotheses, and force a decision between them. *Either* we may reject the "All" of totality while *preserving* the "One" of consistency – this is Badiou's solution – or, alternatively, we may preserve the All of totality while *sacrificing*, at least in certain cases, the One of consistency. This alternative, as I shall demonstrate, essentially defines the possibility of a different theoretical/critical orientation, one which certainly shares with Badiou's "generic" orientation his essential rejection of both constructivism and traditional metaphysics, but is nevertheless capable of underlying very different critical positions and results.

II

In a suggestive chapter from his 1998 book *Briefings on Existence*, Badiou describes what he sees as three possible "orientations in thought."[19] In each of the orientations, as Badiou notes, what is at stake is the relationship of thinking to being itself, the relationship famously named by Parmenides in the assertion that "The Same is there both for thinking and for being" or that "being and thinking are the same."[20] Each 'orientation,' then, regulates this relationship, or this possibility of thought to comprehend the infinite totality of being, by authorizing in different ways the inscription or assertion of existence:

> I call an "orientation in thought" that which regulates the assertions of existence in this thought. An orientation in thought is either what formally authorizes the inscription of an existential quantifier at the head of a formula, which lays out the properties a region of Being is assumed to have. Or it is what ontologically sets up the universe of the pure presentation of the thinkable.[21]

[19] Alain Badiou, *Briefings on Existence: A Short Treatise on Transitory Ontology*, trans. Norman Madarasz, SUNY Press, [1998] 2006, chapter 2.

[20] Badiou (*Briefings on Existence*), p. 52.

[21] Badiou (*Briefings on Existence*), p. 53.

Since each orientation thus preconditions the thinkability of being as a whole, we may indeed take them to amount to a series of positional total relations to the infinite totality of what is, or what is sayable of it. And then we may see in philosophy a privileged domain of reflection on what is involved in these different ways of being oriented toward being itself, of "setting up" or "laying out" what it means to be.

What, then, are the possible orientations in thinking, understood as possible relations to the totality of being as such, or as sayable? Badiou distinguishes among three. The first is what Badiou calls the "transcendent" orientation:

> The … transcendent orientation works as a norm for existence by allowing what we shall coin a 'super-existence.' This point has at its disposal a kind of hierarchical sealing off from its own end, as it were, that is, of the universe of everything that exists. This time around, let us say every existence is furrowed in a totality that assigns it to a place.[22]

What Badiou terms the transcendent orientation, thus, sets up the totality of beings by reference to a privileged being, a "super-existence" that assures the place of everything else, while at the same time obscuring its own moment of institution or the grounds of its own authority. Thus, the totality is conceived as the determined order of an exact placement of beings, while it is covertly regulated by an exemplary Being, conceived as superlative, transcendent to the order of things, and ineffable in its terms. Here, in a gesture typical of philosophy from Plato up to Nietzsche, the being of norms is assumed in the figure of a privileged, sovereign Being, while the basis of their authority is not further examined. Here as well, infinity is thinkable only in terms of such a sovereign Being, as the transcendence or ineffability of a singular Absolute wholly beyond the finitude of human life and existence, whose excess is simultaneously cloaked with the aura of obscurity. Without further ado, we may appropriate Heidegger's term (and indeed his whole description of it) for this orientation: thus, we term it the "onto-theological."[23]

The second orientation is also one we have already discussed. It is the one that is implicit in traditional nominalism, as well as in some forms of critical thought since Kant, but reaches its full methodological expression only with the twentieth-century linguistic turn. This is the orientation that relates to the totality of what is sayable about Being by means of an explicit tracing of the structure and boundaries of language; Badiou terms it "constructivist":

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[22] Badiou (*Briefings on Existence*), p. 55.

[23] Though the terms in which Heidegger describes the historical structure of thought that determines beings by reference to some one superlative being are thus useful for the current project, I do not treat Heidegger's own "being-historical" critique of metaphysics and presence in any detail here.

> [The constructivist orientation] sets forth the norm of existence by means of explicit constructions. It ends up subordinating existential judgment to finite and controllable linguistic protocols. Let us say any kind of existence is underpinned by an algorithm allowing a case that it is the matter of to be effectively reached.[24]

Here, with the "constructivist" orientation, the totality of the sayable is regulated by the discernable protocols of meaningful language, comprehensible in themselves and capable of distinguishing between the sayable and the non-sayable. Thus, reflection on the (presumably determinate) structure of language yields a kind of critical enterprise that involves the drawing of a regulative line between sense and nonsense, or between the sayable and what cannot (by means of the determinate norms definitive of language as such) be said. In some of its most exemplary forms, this is the project of a kind of limitative *policing* of the sayable; the verificationism of Carnap and Ayer is a prime example. Here, the totality of the sayable is itself understood as comprehended by the determinate syntactical rules for the use of the language in question, and thus as not only a *bounded* but a *finite* whole, outside of which it is possible for the theorist or the inventor of languages unproblematically to stand. The methodological correlate of this orientation is thus the conventionalism that sees the totality of *a* language as wholly perspicuous from outside its determinate bounds, but forecloses or ignores the question of the possibility of *language*, or meaning, *as such*. Since it is always possible to stand outside a determinate language and specify its principles, it is always possible to exceed a determinate, bounded language with another one. Thus, the constructivist orientation can grasp infinity only as the *potentially* infinite openness of a successive hierarchy of types, or meta-languages, each one of which can grasp all of those beneath it, but at the cost of its own possible capture by a still higher language.

Finally, Badiou poses as the third possibility the "generic" orientation that determines his own project in *Being and Event* and elsewhere. This orientation differs from the other two, at least, in insisting upon the relevance of actual and multiple infinities to our understanding of being as such. Arising in this way from the event of Cantor's discovery of multiple infinities, it takes into account (where the other two do not) the radical implications of the representation of the infinite totality within itself, what is figured in the possibility of diagonalization:

> The third orientation posits existence as having no norms, save for discursive consistency. It lends privilege to indefinite zones, multiples subtracted from any predicative gathering of thoughts, points of excess and subtractive donations.

[24] Badiou (*Briefings on Existence*), p. 55.

> Say all existence is caught in a wandering that works diagonally against the diverse assemblages expected to surprise it.[25]

Thus, applying no norm *other than formal consistency*, the generic orientation relentlessly pursues, along the diagonal, the existence of all that which escapes constructivism's limitative doctrine of thought. Indeed, it is one of the most impressive accomplishments of Badiou's *Being and Event* rigorously to formalize both the constructivist and the generic orientations in terms of set theory. Badiou thereby shows how the apparatus of set theory leaves open the possibility, beyond anything constructivism can allow, of the "generic set" which, though real, is completely indiscernible within ontology, and hence also the possibility of the extension of any determinate situation by means of a generic "forcing" of the indiscernible. This is the *coup de force* involved in Badiou's appeal to Cohen, which he takes to authorize the doctrine of the Event that shows the inherent limitation of any constructivist doctrine and which ensures, for Badiou, that there can indeed be a doctrine of the advent of the radically new, beyond what any existing language can possibly figure.[26]

III

Badiou's generic orientation is thus one that takes account of the paradoxical possibility of total self-reference, indeed passing through such self-reference to generate the doctrine of multiple infinities and draw out the transformative consequences of Cohen's technique of forcing. In so doing, though, Badiou takes the generic orientation to refute any critical appeal to the structure or nature of language (which he assimilates uniformly to the constructivist orientation). Does it in fact do so, though? Or is there, in fact, *another* possible method by which thought, figuring the radical paradoxes of self-belonging and totality that find expression in diagonalization, Russell's paradox, and Gödel's theorem, can relate to the totality of what can be said, or of what is?

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[25] Badiou (*Briefings on Existence*), p. 55.

[26] The generic orientation seems to be substantially original with Badiou, but there are important anticipations of its view of Truth as the result of the diagonalization of particular situations, particularly in the views of some of the mathematicians and formalists on whom he draws. The most significant of these is probably Gödel himself, who took his own incompleteness theorems to establish the necessary existence of truths that, although they could not be proven by any formal system, were nevertheless accessible to human mathematical intuition. There are also significant anticipations of Badiou's position in certain pre-WWII philosophers of mathematics, for instance Leon Brunschvicg, and the philosopher and resistance fighter Albert Lautman, who sought in his "Essay on the Mathematical Notions of Structure and Existence" to undertake a "positive study of mathematical reality," drawing on the results of Gödel and the metalogical methods suggested by Hilbert's formalist program. (See Albert Lautman, *Essai sur les notions de structure et d'existence en mathematiques: Les schemas de structure*, Paris, Hermann & Cle, 1938.)

In fact there is another orientation, one that is fully cognizant of these paradoxes and yet does not refuse the relevance of internal linguistic reflection in the way that Badiou's generic orientation does. We have already met it: it is the *paradoxico-critical orientation* that operates by tracing the de-totalizing implications of the paradoxes of self-reference at the boundaries of the thinkable, or sayable. That this orientation is indeed fundamentally different from Badiou's, despite its common passage through the paradoxes of self-reference, is already suggested by the very different relation it bears to the analysis of the structure of language: that is, whereas Badiou's generic orientation (officially at least) positions itself beyond or before all reflection on language and its structure, the paradoxico-critical orientation depends crucially, as we have seen, on the possibility of language self-referentially to figure itself by displaying its own structure (even if this figuring will necessarily be partial and paradoxical).[27]

With this in mind, we can now specify the most basic distinction between Badiou's generic orientation and the paradoxico-critical one. It is this: given the paradoxes that force a choice, whereas Badiou's generic orientation decides *for* consistency and against completeness, the paradoxico-critical orientation is based on the decision *for* completeness and against consistency. Thus, whereas Badiou's generic orientation maintains the methodological aim of consistency at all cost, up to the point of denying the existence of a whole or totality at all, the paradoxico-critical mode typically works by *affirming* the existence of a totality (of all that can be said, or of the world, or of Being) and tracing the contradictions and antinomies that thereby arise at its boundaries. It does *not* necessarily seek a resolution of these contradictions, but indeed finds them to be necessary to the structuration of the relevant totalities that it considers. Thus, by contrast to Badiou's decision against the One, paradoxico-criticism can be considered to be committed to the relentless *affirmation* of the One, regardless of its being constitutively rent by the paradoxes of in-closure at its boundaries. It is in this fashion that it performs its critical work, tracing and documenting the complex topology of in-closure *without* attempting to resolve it into a univocally consistent doctrine of being.

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[27] Thus, Peter Hallward ('Introduction: Consequences of Abstraction,' in Peter Hallward (ed.), *Think Again: Alain Badiou and the Future of Philosophy*, London, Continuum, 2004) perceptively suggests that the relationship between language and inconsistency motivates Badiou in his discussion of "presentation" and "representation": "It is no accident that Badiou is especially careful to circumscribe the most obvious link between what we are and how we are presented, namely language. If fundamentally we *are* speaking beings, and if language is advanced as the most general medium of our presentation, then the rigid demarcation of consistency from inconsistency collapses in advance; it is exactly this consequence that Badiou's steadfast refusal of the linguistic turn is designed to forestall."

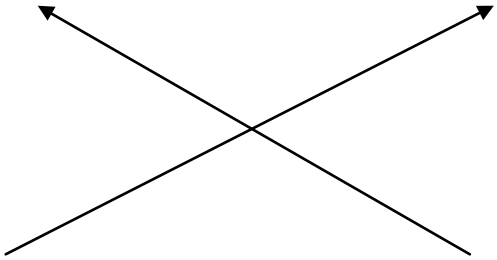
Paradoxes at the limit of thought are contradictions; and I have argued that it is possible for a formal doctrine of limits to describe and operate with such limit-contradictions. If this work is to be possible, however, it is clearly necessary first to reckon with the objection that has, despite antecedents in philosophers such as Hegel and Marx who certainly acknowledge the existence of contradictions, almost universally prevented formal thought from countenancing the contradictions and paradoxes at the limits of thought and language. This is the objection that a single contradiction vitiates the usefulness of any logical system in which it appears, and so that no coherent logical analysis of any contradiction is possible. In a series of works, the logician Graham Priest has argued against this ancient prejudice, which finds expression in of the traditional principle *ex contradictione quodlibet*: "from a contradiction, anything follows." If it were true, indeed, that the existence of a single contradiction could lead to *any* conclusion, then any formal/logical system that includes contradictions would indeed be useless, since it would be of no use in tracking truth or understanding the world. Against this principle, however, Priest argues for the somewhat counter-intuitive doctrine of *dialetheism*. This is the view that there are *true contradictions* – that is, sentences, P, for which both P and not P are true.

In fact, as Priest demonstrates, it is perfectly possible formally to construct a dialetheist logic that tolerates contradictions in *certain* cases without allowing these contradictions to "explode" to the proof of any claim whatsoever.[28] Such a logical structure is, as Priest has also argued in a more recent work, in fact just what we need in order to formalize the contradictions that seem necessarily to exist at the limits of thought and language.[29] Such paradoxes arise directly from the recurrent phenomenon that, in order to speak about the limits of language (or think about the limits of thought) we seemingly need to stand both *within* and *outside* these limits simultaneously: thinking about the totality of the thinkable requires that we stand outside the boundaries, but whatever we thereby think will, being thinkable, again be within them. This general situation – which Priest calls an *inclosure* contradiction – can be seen as underlying, as he argues, a wide variety of problems and puzzles in the history of philosophy from the pre-Socratics to Derrida. With respect to the problematic of totality and limitation that Badiou pursues, its use is that it can effectively be mobilized in a critical mode by practitioners of paradoxico-criticism to model and interrogate the contradictions that characterize the totalities of the sayable and thinkable themselves.

[28] Graham Priest, *In Contradiction: A Study of the Transconsistent*, 2d Edition, Oxford U Press, [1987] 2006, pp. 53-72.
[29] Graham Priest, *Beyond the Limits of Thought*, Second Edition, Oxford, Clarendon Press, 2002.

By arranging the four orientations, we obtain the following schema, which displays some interesting symmetries and relations.

Language captures Truth	**Truth exceeds language**
Paradoxico-Critical: Completeness, inconsistency	**Generic**: Consistence, Incompleteness



Criteriological/Constructivist	**Onto-Theological**

We can also give brief definitions of the four orientations, differentiated according to their attitudes toward the totality of language, the thinkable, or being:

Paradoxico-critical: Any position that, recognizing reflexivity and its paradoxes, nevertheless draws out the consequences of the being of the totality, and sees the effects of these paradoxes always as operative within the One of this totality.

Generic: Any position that, recognizing reflexivity and its paradoxes, denies the being of the totality and sees these paradoxes as traversing an irreducible Many.

Criteriological: Any position that attempts to delimit the totality consistently from a stable point outside of it.

Onto-Theological: Any position that sees the totality as complete and consistent in itself, though beyond the grasp of finite cognition.

On the one hand, the paradoxico-critical orientation is clearly distinct, as we have seen, from the constructivist orientation that seeks to delimit being by means of an investigation of the fixed structure of language. Rather than promote such a limitative doctrine, it takes account of the paradoxes of self-inclusion (which make it impossible

to preserve both consistency and completeness simultaneously) in order to trace the fundamentally paradoxical structure of limits and limitation, up to the paradoxes involved in the fact that language appears in the world at all. A closely related distinction concerns the question of a metalanguage, for instance a language *distinct* from English in which it would be possible to describe the structures of truth and meaning exhibited by the English language itself. As we have seen, the criteriological orientation of Russell and Carnap, which begins by attempting to specify the bounds of a single language by means of a description of its rules, invokes not only one, but indeed a whole hierarchy of distinct metalanguages, each one necessary in order to describe the constituent structure of the one underneath. Paradoxico-criticism, by contrast, refuses to countenance any such metalanguage, affirming (though it may indeed lead to paradoxes) that a natural language such as English bears within itself all the resources (problematic though they may be) for talking about its own constituent structures.

On the other hand, the paradoxico-critical orientation is also distinct, as we have also just seen, from the generic orientation. In particular, these differ fundamentally in how they consider the status of totality: whereas the generic orientation saves consistency by denying completeness, the paradoxico-critical orientation *affirms* an inconsistent totality, documenting the inconsistencies that inherently arise when language ventures (by a necessity of its own structure that can hardly be denied) to speak the whole as One. In some recent remarks on Badiou that criticize mildly his formulation of the generic orientation, and suggest the elements of an alternative, Slavoj Žižek emphasizes the way in which this paradoxico-critical thinking of the One necessarily differs from the emphasis on multiplicity that Badiou's generic orientation and other orientations of contemporary thought share:

> What the … extolling of multiplicity is missing is the noncoincidence of the One with itself, the noncoincidence which makes the One the very form of appearance of its opposite: it is not only that the complexity of its situation undermines every One – much more radically, it is the very oneness of the One which redoubles it, functioning as the excess over the simple one. The function of void is crucial here: what explodes every One from within is not a complexity which subverts its unity, but the fact that a void is a part of every One; the signifier-One, the signifier unifies/totalizes a multiplicity, is the point of inscription into this multiplicity of its own void. This is why every name is ultimately tautological: a 'rose' designates an object with a series of properties, but what holds all these properties together, what makes them properties of the same One, is ultimately the name itself. Consequently, the One as the "empty

> signifier" is the point at which, as Lacan put it, the signifier falls into its signified.[30]

As Žižek suggests, the ultimate point of the paradoxico-critical orientation is not really to insist upon the One rather than the many, but rather to show how the most rigorous One essentially *becomes* many as soon as it passes through the "unifying" function of language, thus producing the gulf between the sign and its reference.[31] This radical gulf, present and unforeclosable beneath every ordinary use of language, is figured by the paradoxes of self-inclusion and self-reference that occur at the point of the manifestation of language itself, the point of a necessary indistinction between signifier and signified, where the very logic of language is manifest syntactically. This problematic point, traced variously in the diverse structuralisms and systems of the twentieth century, locates both the system's inherent excess and its possible disruption. Here, it is apparently possible to speak, as Derrida once did, of an *event* consisting in the "rupture" and "displacement" of structure as such, at the point of redoubling that is also its very core.[32] It remains to be seen how we should understand the relation of this kind of linguistically defined event to Badiou's kind, stripped from any relation to language and verified by the uncompromising formalism

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[30] In Adrian Johnston, *Badiou, Žižek, and Political Transformations: The Cadence of Change*, Northwestern U. Press, 2009, pp. 192-93.

[31] Compare, also, these critical remarks on Badiou, in which Žižek seems to occupy, very clearly, the position of paradoxico-criticism with its denial of a metalanguage position and its constitutive assertion of the 'internal' gap introduced by the One's relation to itself: "…there is a Kantian problem with Badiou which is grounded in his dualism of Being and Event, and which needs to be surpassed. The only way out of this predicament is to assert that the unnameable Real is not an external limitation but an *absolutely inherent* limitation. Truth is a generic procedure which cannot comprise its own concept-name, a name that would totalize it – as Lacan put it, 'there is no meta-language' (or Heidegger: 'the name for a name is always lacking') and this lack, far from being a limitation of language, is its positive condition. It is only because and through this lack that we have language. So, like the Lacanian Real which is not external to the Symbolic but rather makes it non-all from within (as Laclau puts it: in an antagonism, the external limit coincides with the internal one), the unnameable is inherent to the domain of names…The true materialist solution is thus that the Event is *nothing but* its own inscription into the order of Being, a cut/rupture in the order of Being on account of which Being cannot ever form a consistent All." Accordingly, Žižek says, "we should assert" from a Lacanian position that "the ultimate ontological given is the gap which separates the One from within." (Slavoj Žižek, 'From Purification to Subtraction: Badiou and the Real,' in Peter Hallward (ed.), *Think Again: Alain Badiou and the Future of Philosophy*, London, Continuum, 2004, pp. 178-79.)

[32] "Perhaps something has occurred in the history of the concept of structure that could be called an "event," if this loaded word did not entail a meaning which it is precisely the function of structural-or structuralist-thought to reduce or to suspect. But let me use the term "event" anyway, employing it with caution and as if in quotation marks. In this sense, this event will have the exterior form of a *rupture* and a *redoubling*." (Jacques Derrida, 'Structure, Sign, and Play in the Discourse of the Human Sciences', in Alan Bass (ed.), *Writing and Difference*, trans. Alan Bass, London, Routledge, 1966, p. 278.)

of mathematics itself. But such, it now seems, are the stakes of the still poorly understood *fact* of the appearance of language in the world, and of the questions of formalism and action that problematically manifest it.

The two orientations at the top of the diagram both thus have it in common that they result from *differing* reactions to the paradoxes of total self-inclusion; in this respect they are distinct from the two orientations at the bottom, which must both, thus, be considered to be pre-Cantorian in maintaining the possibility of jointly preserving consistency and completeness. However, the two orientations on the left also share something, despite being respectively pre- and post-Cantorian in these respects; in particular, both share a critical motivation grounded in reflection on the structure of *language*. For both of these orientations, it is necessary, in understanding the possibility of speaking being at all, first to pass through (and do we ever emerge?) a deep reflection on language and its formal structure; it is in this way that they both figure the relationship of formalism to what is. For the two orientations on the right-hand side, Badiou's generic orientation as well as the traditional onto-theological one, by contrast, the structure of language ultimately determines neither what *is* nor what can *appear*; whatever is consequent upon the structure of language *per se* is itself secondary to the existence of beings and truths which may transcend or escape it. Of course, Badiou's generic orientation is not thereby equivalent, either, to the onto-theological doctrine of transcendence; whereas the (pre-Cantorian) orientation of onto-theology lodges truth in the privilege of a singular, obscure and transcendent super-Being, Badiou's generic orientation sees truth only in the infinite procession of multiplicities, without end or higher synthesis.

Thus we may group the two orientations on the left as *critical* doctrines of language; whereas those on the right are *dogmatic* doctrines of truth. (I do not mean this term to be pejorative, here, but simply to indicate the point of their common *insistence*: that there must be some truth beyond language, whereas the orientations on the left are linked in refusing to consider truth outside its possibility for linguistic expression, however this possibility may manifest itself.)

Finally, there are also revealing connections along the diagonals. The diagonal from constructivism to the generic represents the common norm of *consistency*. This is as much a norm for (for example) Carnap's constructivism as it is for Badiou's relentless pursuit of mathematical structures; it is marked in both in the absolute privilege of logical rules and the assumption that language, in order to discern a realm of Being, must maintain its consistency at all points. From onto-theology to paradoxico-criticism, on the other hand, we may draw the line of *totality* (or completeness); for both orientations involve the assertion of an actually existent whole.

This is evident, for instance, in the very direct way that paradoxico-criticism interrogates the position of the sovereign Being that assures the order of the totality within onto-theology; for in order to interrogate the force and authority of such a sovereign, it is necessary first to acknowledge and then to interrogate its actual relationship to the whole (of which it is, invariably, also an element). Just as profoundly, the diagonal line of consistency that links constructivism to the generic orientation denies or forecloses the existence of the totality by asserting the non-all, whether in the form of constructivism's infinite open hierarchy of metalanguages or the generic orientation's infinite procession toward the multiplicity of truths. The point of crossing of the two lines is, once again, the paradox of self-inclusion (in its Cantorian, Russellian, or Gödelian forms), which makes it impossible to preserve consistency and completeness simultaneously.

More generally, it may be possible to describe philosophical/political thought about signs and meanings, finitude and infinitude, as today standing at the junction of a *critical either-or* between the two post-Cantorian orientations, that of the generic and the paradoxico-critical. The two orientations touch on almost every important question of contemporary political theory, but they can be traced in terms of their divergent responses to a common beginning: the fixed point of the symbolism of self-reference. If this is right, then Badiou's radical project, situated in the larger critical context explored here, also suggests a radical return to the deep and profound questions of the formal symbolism of reflexivity, up to the possible formalization of the structure of subjectivity itself.

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